Mathematics JAN 16 1939 D-26

THE MATHEMATICS TEACHER

Volume XXXII JANUARY 1939 Number 1

CONTENTS

		PAGE
The Mathematics of Elementary Chemical Calculations	Charles F. Rogers	3
The Challenge of the Slow Pupil	Harry Eisner	9
When Wishes Come True—A Mathematics Play	Hannah A. Parkyn	16
A Child's Concept of Numbers	George H. Whiteaker	25
The Art of Teaching Teaching the Law of Signs in Multiplication	Elbert Fulkerson	27
Twentieth Annual Meeting		30
Editorials		33
In Other Periodicals		35
News Notes		37
New Books		40

PUBLISHED BY THE
NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS
MENASHA, WISCONSIN : NEW YORK, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools

Editor-in-Chief—William David Reeve, Teachers College, Columbia University.
Associate Editors—Vera Sanford, State Normal School, Oneonta, N.Y.
W. S. Schlauch, School of Commerce, New York University.

OFFICERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

President—E. C. Christopherson, Miami University, Oxford, Ohio.
First Vice-President—J. T. Johnson, Chicago Normal College, Chicago, Ill.
Second Vice-President—Ruth Lane, University High School, Iowa City, Iowa.
Secretary-Treasurer—Erwin W. Schreiber, Western Illinois State Teachers College, Macomb, Ill.
Director of State Representation—Florence Brooks Miller, 3295 Avalon Road, Shaker Heights, Ohio.

ADDITIONAL MEMBERS ON THE BOARD OF DIRECTORS

One Year
H. R. Barrell, Chicago, Ill. 1939
Virgil Mallery, Montclair, N.J. 1939
Leonard D. Haertter, Clayton, Mo. 1939

Two Years
Wm. Betz, Rochester, N.Y. 1940
Martha Hildebrandt, Maywood, Ill. 1940
Edith Wolley, Minneapolis, Minn. 1940

Three Years
Kate Bell, Spokane, Wash. 1941
M. L. Hartung, Columbus, Ohio 1941
R. R. Smith, Springfield, Mass. 1941

The National Council has for its object the advancement of mathematics teaching in elementary and secondary schools. All persons interested in mathematics and mathematics teaching are eligible to membership. The membership fee is $2 per year and entitles each member to receive the official journal of the National Council—The Mathematics Teacher—which appears monthly except June, July, August and September.

Communications relating to editorial matter, subscriptions, advertisements, and other business matters should be addressed to the office of the

THE MATHEMATICS TEACHER
525 West 120th St., New York City. [Editorial Office]

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $2.00 PER YEAR (*eight numbers*)

Foreign postage, 50 cents per year; Canadian postage, 25 cents per year. Single copies 30 cents. Remittances should be made by Post Office Money Order, Express Order, Bank Draft, or personal check and should be payable to THE MATHEMATICS TEACHER.

PRICE LIST OF REPRINTS

	8 pp.	9 to 12 pp.	13 to 16 pp.	16 pp. 17 to 20	20 pp. 21 to 24	24 pp. 25 to 28	28 pp. 29 to 32
		$4.85		$4.60	$ 5.55	$ 9.85	$11.40	$12.15
		1.00		7.40	10.35	12.50	14.25	15.30
	1.00	1.00	2.65	3.35	4.60	4.70	5.75	6.50

[illegible] deduct 5%; for [illegible] cents or more deduct 10%.

[illegible] 50 covers [illegible] additional 50 copies, $1.00; additional [illegible] each. Printed on one side [illegible] 20 plates, $1.00; additional [illegible] each.

[illegible] more than 32 pages add [illegible] to make total. Illustrated: for 44 pages add [illegible] 52 pages add [illegible] 12 pages.

[illegible] any portion of [illegible] reproduction or alteration in text or cover, an [illegible] will be made.

[illegible] THE MATHEMATICS TEACHER [illegible] advertisements.

Math Ed
Watts

THE MATHEMATICS TEACHER

Volume XXXII Number 1

Edited by William David Reeve

The Mathematics of Elementary Chemical Calculations

By CHARLES F. ROGERS
Division of Agricultural Biochemistry, University Farm, St. Paul, Minnesota

When the pale moon hides and the wild wind wails,
And over the tree tops the night-hawk sails,
The lone wolf sits on the world's far rim
And howls. And it seems to comfort him.
—Anonymous

THE COURSES and currents in many contemporary educational pronouncements resemble a mature river, which is full of eddies, uncertainties, shoals, meanderings, and new channels, and which has occasional snags both visible and invisible. Some of these snags can cripple almost any craft that encounters them.

On all sides of this stream, which seeks the easiest path toward an as yet imperfectly defined goal, lie fields of knowledge. Some of these fields have been worked over so long and their basic contents are so well understood, that for all ordinary purposes they may be considered to be permanently fixed in the fact requirements for their understanding and interpretation. In the lowlands nearest the stream are the elements of mathematics, physics, chemistry, biology, and any other field where knowledge has become exact and reproducible and which are less liable to changing significance or fact content than are the uplands of higher intellectual attainment, which are still being actively explored.

It is natural that these lowlands of elementary subject matter are most in danger of being submerged under floods of Educational Evangelism. Manner of presentation of a subject to neophytes may vary with the shifting sands of opinion, but adequate elementary knowledge of a subject-matter field implies the availability and facile use by the neophyte of those tools by which the subject can be most conveniently and rapidly handled.

The American Chemical Society has endorsed the report of its Committee on the High School Teaching of Chemistry[1] which finds much inadequacy in the teaching of science in high schools to be due to insufficiently experienced teachers. The corrections suggested by this committee point in the right direction. Williams[2] has ably justified the teaching of mathematics both as a cultural and as a disciplinary subject for those who are able to meet its demands. He also stresses its universal value when intelligently applied to everyday problems, as well as to those that seem to be removed from immediate human-need significances.

Much of chemistry becomes an exact and serviceable science as it becomes quantitative. Its quantitative nature is demonstrable only in numerical terms. The mathematical needs of the most com-

[1] *School and Society*, 43: 604–606. No. 1114. May 2, 1936.
News Edition, Ind. Eng. Chem., 14: 147. April 20, 1936.

[2] Williams, K. P., "Why we teach mathematics." *The Mathematics Teacher*, 29: 271–280. No. 6. October, 1936.

plicated volumetric and gravimetric quantitative chemical analyses rarely exceed the solving powers of arithmetic.

Because of the essentially numerical nature of even the most elementary adequate presentation of chemical facts and ideas, it might be helpful to restate the simple mathematical requirements of elementary chemistry, so that those who teach the beginnings of mathematics in the grade and high schools will have refreshed for them what at least some of their pupils will later find useful from the mathematics which they are taught in precollege years.

After accuracy in the correct copying of figures and the errorless handling of numbers, the greatest need among present day college students of chemistry is an automatic facility with the simple arithmetic processes which would include the nature and use of fractions: simple, compound, and decimal.[3] Perhaps because human cupidity ordinarily follows arithmetic progression, rather than geometric, addition is the first and greatest desire of those who must attempt to solve problems. The need for addition is quickly detected. Comprehension of the nature of the problem will justify or dispel addition or subtraction.

Each different kind of atom has a different weight per atom. Individual atoms cannot be handled. Large numbers of them are taken together to represent unit quantities. Comparison of these quantities according to the nature of the chemical reactions demands a working knowledge of the denotation, functions and implications of common denominators, more frequently called "same terms." "Common denominators" inspire the awed look in present day students, due to their lack of familiarity with the more powerful mathematical terminology. The proper explanation of the *chemical* significance of the common denominator is the privilege or responsibility of chemistry teachers, but some of them feel that the knowledge of the meaning and operations of the common denominator whether as a part of a fraction or as a means of comparable expression, should be one of the beginner's usable tools when he is digging in the chemical field.

Chemically defined units of elements or compounds are too large to be conveniently or accurately employed in measurements for either teaching or research purposes. Small fractions of unit quantities must be measured, from which the magnitude of the chemical units is computed. In such calculations the student may be forced to take the reciprocals of numbers, reduce a simple fraction to its decimal equivalent, transform quantities to a common basis (denominator) and express the results of an analysis in terms of the parts per hundred (percentage), or in parts per million. He may find it necessary to compute the content of a whole sample from an aliquot, or reduce equivalent fractions to unity, and compute a percentage. (May the chemistry teacher help him? He will usually have to!)

The mathematics of each of the above-indicated steps in such a process is always simple, and usually understandable to all, but when some or all of these operations must be performed on one innocent problem, by a suffering student, the whole process becomes horrendous. The modern student makes a frightened diagnosis in his search for the answer, and fumbles with his fundamental instruments which are unfamiliar to him and dulled by disuse or even misuse. He needs most of all self-control and powers of calm analysis to take him as far as his comprehension will permit.

Of reciprocals most students have never heard. It would seem that this is a part of the knowledge of fractions. Decimal equivalents of any fraction present occasional difficulties. When they become factors or divisors, they take on some peculiar property which makes their use seem to be different from numbers made up of the same figures, but with the deci-

[3] Rogers has recited students' handicaps because of unfamiliarity with these elementary processes. See "Arithmetic and emotional difficulties in some university students." *The Mathematics Teacher*, 30: 3–9. No. 1. January, 1937.

mal point to their right. These troubles are most apparent when one must find the numerical size of some unit quantity. Almost always when an expression like, "0.04 chemical units of sodium = 0.920 gr. sodium," confronts a student, the proportional urge blinds him to the simplest of devices for the reduction to unity, which is: "1.00 chemical unit of sodium = 0.920 gr. sodium/0.04 chemical units = 23.0 gr. of sodium." This operation is perhaps as difficult to make clear to students as all other arithmetical processes they must use. Although certainly new to most modern college students, it was orthodox arithmetic in grade schools before and during the last great war, and is still inescapable in modern chemistry.

Along with the need for instruction in such practices which should be presupposable by chemistry teachers, comes another—aliquot parts. To be sure, aliquot parts are of less importance than some other devices, yet as an application of fractions, their presentation would protect the future student against his own vacuous look when he is asked, "What aliquot is 100 of 1000?" It would also protect him against completely misunderstanding written directions.

Of all arithmetic expressions, percentage is probably the most used. College chemistry instructors must hammer continuously on the calculations and significances of percentages. Even though problem statements always explicate the bases of percentages to be expressed, it is necessary to point out repeatedly for each individual problem what "is 100%." Conversely, calculation of 100% from some given percentage, which *is a repetition of reduction to unity*, is one of the most difficultly understandable processes of the simple calculations used in ordinary quantitative chemistry.

It is most surprising that percentage gives the general trouble that it does. Some widely used, recently written arithmetic text-book series devote as much as forty per cent of the total pages in the books from grade six up, to use or calculation of percentage.

Compound fractions are needed in some chemical calculations. Occasionally it is necessary to clear fractions and to use the decimal value of symbolized fractions which are called "factors"—another menace. Factors may be multiplied together, or their reciprocals used to produce a derived factor from which some of the terms in the symbolic factors are cancelled out. Instantly comes the question, "If those intermediate terms are cancelled out why must we use the numbers for those cancelled fractions in the figuring of the factor?" Numerical demonstration is the only conclusive proof that can be offered, but it does not convince students to be *told* that the interior fractions have to be used when only their ghosts remain in the final expression.

An example:

From the chemical factors:[4]

$$2Al \text{ to } 2AlPO_4 = 0.2211$$
$$2AlPO_4 \text{ to } P_2O_5 = 1.7176$$
$$P_2O_5 \text{ to } Mg_2P_2O_7 = 0.6360$$
$$Mg_2P_2O_7 \text{ to } 2MgCO_3 = 1.3205$$
$$BaSO_4 \text{ to } MgSO_4 = 1.9390$$
$$BaSO_4 \text{ to } Mg_2P_2O_7 = 1.0812$$

Calculate the chemical factors:[4]

$$Al \text{ to } MgCO_3$$
$$Mg_2P_2O_7 \text{ to } Al$$
$$P_2O_5 \text{ to } BaSO_4$$

The chemical conditions of equivalence already have been satisfied in the calculation of these factors, so that they can be used as fractions without further treatment.

The factor Al to $MgCO_3$ takes the form $Al/MgCO_3$. One must choose and multiply fractions until the desired numerator and denominator remain alone. Four fractions are required:

[4] These "factors" are ratios. "Factor" is a chemical term. It is better than "ratio" because it does not suggest proportion. The ratios are of the chemically equivalent molecular quantities of the compounds whose formulas are given.

$$\frac{2Al}{2AlPO_4}\times\frac{2AlPO_4}{P_2O_5}\times\frac{P_2O_5}{Mg_2P_2O_7}\times\frac{Mg_2P_2O_7}{2MgCO_3}=\frac{2Al}{2MgCO_3}$$

$$0.2211\times1.7176\times0.6360\times1.3205=0.319=\frac{54}{168.64}\cdot$$

What puzzles the students most is that the two interior fractions, which are completely eliminated from the algebraic expression must appear in the arithmetic product.

The second factor to be computed requires the use of a reciprocal fraction whether it is obtained from the given data, or whether the newly calculated factor is taken. Using the newly computed factor:

$$\frac{Mg_2P_2O_7}{2MgCO_3}\times\frac{1}{\frac{2Al}{2MgCO_3}}=\frac{Mg_2P_2O_7\times2MgCO_3}{2MgCO_3\times2Al}=\frac{Mg_2P_2O_7}{2Al}=$$

$$=1.3205\times\frac{1}{0.319}=\frac{1.3205}{0.319}=4.15\cdot$$

It is the difficulties of the mechanical handling of fractions as much as the need for the taking of chemically equivalent quantities that give the students arithmetic headaches.

Substitution and use of equals in their different forms is an algebraic process. Experience alone makes it possible to decide wisely what form to choose. When statements of equalities are given and substitution is made with explanation of the nature of the equalities, there should be no question of the justification for such mathematics.

The measurement of the volume of a solution too large for convenient volumetric handling is possible, when its concentration is determined, and the effect of a measured volume increase on the concentration of the solution is ascertained. Further dilution to any desired concentration is calculable, as is the original volume. An example shows the applications of algebra to the calculation of elementary chemical processes.

A solution of unknown and unmeasurable volume was found to have a concentration of 4.050 grams per liter. One liter of water was uniformly mixed with the solution, after which the concentration was 4.025 grams per liter. What was the volume of the original solution, and how much water must be added to bring the concentration to 4.000 grams per liter?

It must be assumed that the quantity of solute is constant, wherefore the product of concentration by volume is constant and identical for all three concentrations.

Let L_0, L_1, and L_2 be the original volume, and the volumes after the first and second dilutions respectively.

Let C_0, C_1, and C_2 be the corresponding concentrations.

Then $C_0L_0=C_1L_1=C_2L_2=$ total grams of solute, $=G$. (1)

The fact that with a fixed weight of solute, concentration varies inversely with the volume, is stated algebraically:

$$\frac{C_0}{C_1}=\frac{L_1}{L_0}, \qquad (2)$$

a fact which could have been ascertained algebraically from the form of statement (1).

The problem is now in a form suitable for treatment. The student should seek to develop a general expression for the changes of concentration with respect to the volume changes. He knows that a volume change will produce an opposite concentration change of the same relative magnitude. Like the form (2), this too is an inversely proportional relation, so that if the original concentration, C_0, were to be referred to the concentration change, C_0-C_1, it would take the form

$C_0/(C_0-C_1)$, and would equal the relation of the second volume, L_1, to the volume change, L_1-L_0.

The equality,

$$\frac{C_0}{C_0-C_1}=\frac{L_1}{L_1-L_0}, \qquad (3)$$

would immediately give the value of L_1 when C_0, C_1 and (L_1-L_0) are known. Substituting values given in the problem, this statement becomes:

$$\frac{4.050}{4.050-4.025}=\frac{L_1}{1.00\text{ liter}} \qquad (L_1-L_0=1.00\text{ Liter})$$

$$\frac{4.050\text{ grams per liter original concentration}}{0.025\text{ grams per liter change in concentration}}=L_0+1.00\text{ Liter}$$

$$162=\text{the number of liters in } L_0+1.00\text{ Liter.}$$

$$L_0=162-1=\mathbf{161\text{ Liters}=\text{original volume.}}$$

Equation (1) states that $C_0L_0=G$, wherefore 161×4.050 gr./L. $=652.05$ gr. of solute, which is a fixed quantity. By the same equation,

$$C_2L_2=C_0L_0=G=652.05\text{ gr. of solute,}$$

$$\text{and } 4.000\times L_2=652.05\text{ gr. solute}$$

$$(C_2=4.000\text{ gr./L.}).$$

The final volume, L_2, $=(652.05/4.000)=$ **163.0125 Liters,** the volume necessary to which to dilute the solution in order to produce a solution of 4.00 gr./L.

Expressions (1), (2), and (3) are accepted mathematical shorthand, which the student should develop for himself with each problem rather than accept upon authority for later indiscriminate use. For this reason these expressions are not *called* formulae within the hearing of students. The chief reasons for this refusal is faulty memories of students, and the inapplicability of these formulae to problems that do not fit into the specific mould. Almost all of the fundamentals of volumetric quantitative analysis are involved in this problem. Under other sets of circumstances, the fundamentals take different forms and names. This fact is another excellent reason against allowing students to try to absorb and use mathematical formulae. As one of the more involved problems beginning students would encounter, the above problem is an example of what it is desirable for students to be able to do. They should know how to set up the problem in calculable form, to state the given or assumed relationships in suitable symbols or directly in numerical terms if they prefer, and to recognize what the new numbers mean when they have been obtained by arithmetic processes.

One can easily get lost in this problem unless he properly labels every step and every number *to be* obtained by calculation. As much depends upon the names of the numbers or quantities for the proper solution of problems, as upon understanding the processes which must be employed. Until problem solving has become a habit with students before they enter the university, a true idea of the sad plight of students who have never faced problems which require analysis, can be had only by a visit to a place of so-called higher learning when problem solutions are being explained.

Simultaneous algebraic equations are potent and necessary in a few calculations of a relatively limited kind of elementary analyses, but for the less pretentious courses such problems and analyses are avoided.

Anyone who has left a subject unused for a while knows how quickly it gets away from him, unless he has thoroughly mastered its basic concepts and processes. He finds himself in doubled difficulties when he tries to use tools with which he previously only fumbled. He is seriously

hampered in his work on some strange, new matter in which he is expected to know what he is doing, to have at hand facilities with which to *work*, and with which he does not have to tinker.

The chief trouble with students lies in their own lack of experiences in which they were forced to face and master even simple problems, when those problems were not set up for them ready for computation.

Probably because they have been supplied in the past, the pressure for formulae by which to solve problems is felt by the students, who, with perfect elasticity, transmit this pressure to their instructors. Dependence on given formulae for such simple flexible conditions as have been discussed here, is mental invalidism, which is likely to result in more errors than would otherwise occur. Faith in a formula whether or not it is false for conditions which it seems to fit, will destroy desire to exercise judgment.

One of the most treacherous of the formulae for problem solution is the proportion. Proportion is a useful, powerful, often clumsy device, which is an arithmetic syllogism. Its conclusions are only as true as the original premises. Unfortunately for the proportion, the order of statement of the premises affects the verity of the conclusions. As an equality of ratios, proportion is a little less dangerous than when it stalks untamed through the young student's illogical jungle of unassimilated facts.

Ratios are fractions. Their properties as fractions have been suggested already. Needless to say, ratios whether as indicated divisions or as numerical values, such as the factors manipulated above, play a larger part in chemical calculations than any other single mathematical operation.

The idea of variation of one quantity with another is a part of the last problem shown in this paper. Variation often goes with ratios. It should be understood both in its algebraic forms and in its arithmetic computations. In the higher mathematics, with which this paper does not deal, variables are one of the most important mathematical concepts.

More than any single specific tool, proper working of elementary fields of science requires the capacity to analyze a situation, to set down in logical order—that is, in workable relations—the facts given, or known, and to proceed from old knowledge to new knowledge to the solution of the problem presented. Those who have developed a problem-solving technic have immense advantage over others who fearfully scratch numbers in any order they can imagine in a terrified hope for an answer. It is these latter who cry aloud for formulae, and cleave to proportion, when clear thought would show that half a proportion was more frequently better than one whole proportion.

There are those who fully believe in the value of mathematics who will remind the author that application of mathematical tools to specific problems is more difficult than the abstract mathematical manipulations of numbers or symbols. The author hastens to assert his own painful awareness of the fact. He believes, however, that he should be expected only to burnish the mathematical tools of students and to show students how and why these tools apply to chemical problems, rather than to have to try to teach some chemical calculations *after* the justification and mechanics of simple arithmetic concepts have consumed most of his and the student's allowed time in the term. He believes that much of what he now has to teach was at one time drilled into students in grade schools, but it was not drilled into most of those now in college. Technic in problem solving is perfected by practice. It should have been begun before college entrance was permitted. Why should college credit be given for making up deficiencies so elementary and so fundamental that grade and high schools should have at least partly overcome them?

The Challenge of the Slow Pupil*

By HARRY EISNER
Manual Training High School, Brooklyn, New York

THE IMMENSE growth of the secondary school population throughout the country during the past twenty years is a fact of universal knowledge. In New York City, senior high school registration has increased from about 60,000 in 1917 to 250,000 in 1937. This tremendous increase is composed largely of pupils from the lower brackets of intelligence who find themselves completely submerged by the traditional mathematics and are failing in vast numbers. Probably no problem in the field of the teaching of secondary school mathematics is more pressing for early relief or solution than the problem of providing an appropriate education for the so-called slow pupils.

Of course the word "slow" as applied to high school boys and girls is a descriptive adjective that has relative significance only, since the term directs attention upon a group whose mental ability and scholastic performance are of a distinctly lower grade than that of the groups which may be characterized as normal or bright. Furthermore, since the mean intelligence level of the high school population is considerably lower now than it was twenty years ago, it is evident that the present appellation "slow" consigns the recipients of this epithet to a very much lower place in the ladder of mental ability and accomplishment than was the case two decades ago. For the sake of definiteness in this address, I shall identify slowness with comparatively low general intelligence, and define as "slow pupils" those who comprise the lowest 25% in intelligence, as measured by I.Q., in an average heterogeneous high school population. The maximum I.Q. in such a group would be roughly .93. While I realize that the correlation between general intelligence and achievement in mathematics is far from perfect, it must be borne in mind that some pupils possess compensating qualities of earnestness and industry that more than make up for mental shortcomings, and that many pupils of considerable intelligence fail in mathematics through lack of application. It seems a fairly safe assumption that "slowness," or inability to understand and apply mathematical concepts and relationships is a function of general intelligence.

The United States is unique among the nations of the world in its espousal of and adherence to the ideal of universal secondary education. But universal education is far from meaning common or identical education to all. There is, unfortunately, quite a large conservative element within our ranks, some in high places, which clings tenaciously to the standards maintained in the days when the high school population was comparatively select, and refuses to modify subject matter or method of a by-gone day. According to this group, dullards have no place in high school and they should not be permitted to take any mathematics.

In contrast with this uncompromising position there have sprung up during the last ten or fifteen years honestly intentioned attempts at adjustment of mathematical education to fit slow pupils. In some schools, the courses of study are the same for all types of students, but the slow pupils are allowed more time than the others to complete the work. This practice was discontinued by edict in New York City about ten years ago. In other schools adjustment has been attempted by simplifying the courses of study by omitting some of the more difficult topics and eliminating all but the

* Address given at the Joint Meeting of the National Council of Teachers of Mathematics and the Department of Secondary Education of the National Education Association, Monday, June 27, 1938 in New York City.

easiest exercises. In still other schools the modification has taken the form of a shortened course consisting essentially of the arithmetic and intuitive geometry taught in the elementary school. All of these attempted solutions are inadequate because they are based on the fallacy that so-called academic mathematics is good for all high school pupils, and on the further false assumption that all that needs to be done is to make an adjustment in time, or in amount or in degree of difficulty.

Adjustments have been attempted in method as well as in content. In some schools rote mastery has become the chief objective because of the conviction that these slow pupils are quite incapable of any thinking. As a result, drill has assumed in these quarters a position of supreme importance, quite overshadowing comprehension or appreciation. Rules, formulas or procedures are handed down to the pupils by decree; it is the children's responsibility to memorize the laws and the illustrations supplied by the teacher and apply them to precisely similar exercises. In other schools, in their desire to give exercise to the motor skills, teachers have overstressed the constructional aspects of geometry to the extent of almost completely divorcing them from their mathematical significance or interpretation. These instances of pseudo-adaptations of method may be legitimately attacked as stultifying mathematics teaching and revealing a lack of knowledge of the true characteristics of slow pupils.

What then has mathematics to offer to these pupils that will be of genuine value to them and by what methods may the offering be best assimilated by them? Before undertaking the ambitious task of answering this question, let me try to give you a picture of the physical, emotional and mental characteristics of these slow pupils as interpreted by psychologists and confirmed by my own experience.

Physically, these pupils approach very closely normal boys and girls of equivalent age. Their motor coordinations and skills are approximately normal also. Emotionally, too, they react very much as normal boys and girls do. It must be remembered, however, that many of them are two to three years older than normal progress pupils in the same grade, and that their feelings are accordingly on a more adult level. Intellectually, a very significant and pronounced difference is apparent between slow and normal pupils. This difference has been pointed out by leading psychologists as a quantitative one rather than a qualitative one. The slow student can memorize, but does so more slowly than the normal student. He can see relationships provided they are more obvious than they need be for normal children. He can generalize, provided the specific instances are more numerous and the generalizing inference is a short step. He can reason moderately, if the elements in the chain of thought are concrete and comparatively few. He can understand what he reads, provided the vocabulary and the thought content are at a sufficiently simple level, and ample time is allowed for comprehension. With respect to interests, the slow pupils, by reason of their greater age, are more attracted by adult activities and pursuits than are the more gifted but chronologically younger students.

Thus, broadly and roughly sketched, are the characteristics of the slow pupils in our secondary schools. There are hundreds of thousands of them in the country; 60,000 in this city alone. Many of these children leave high school after the first or second year. While in school they present a serious challenge to every professionally minded teacher of mathematics. What should be done with them?

May I offer for your thoughtful consideration the following general principles which reflection and experience have led me to advance as guide posts:

1. Mathematics has something of value to offer in the mental development of every high school boy or girl, irrespective

of the degree of his intelligence. No pupil should leave high school without having had at least a one-year course of appropriate mathematics.

2. No adequate program can be planned or put into execution for slow pupils unless they are grouped in approximately homogeneous classes.

3. One educational aim of a course of study for slow pupils should be to give them an appreciation of the rôle of mathematics in modern life with reference to both quantitative aspect and form.

4. Another educational aim of such a course should be to develop in these slow pupils such concepts, skills and attitudes as will enable them to interpret and utilize mathematics in accordance with their needs in present and future common life situations.

5. The content of the course must not be above the mental level of slow pupils.

6. The methods of teaching must be appropriate to the content and to the mental characteristics of slow pupils.

In the time which remains I shall try to elaborate on these principles.

I am glad to report that the preparation of a modified course of study for slow pupils is now engaging the attention and efforts of a considerably large number of teachers and supervisors. This is a healthy sign and portends much good. In this city the First Assistants' Association, whose members are heads of the various departments in the high schools, is busily at work on a course for non-academically minded pupils in all subjects. The Association of Teachers of Mathematics in New York City, through its Curricula Committee, of which I have the honor to be the chairman, is also preparing such a course. A tentative outline of the latter course will soon be published in the official journal of the Association of Teachers of Mathematics. May I take a little time to describe its main features?

The content of our proposed course is centered about four large topics. The first of these topics is "The Social Uses of Arithmetic" and includes recreational and leisure activities, health activities, thrift activities, investments, keeping and interpreting accounts, paying taxes, providing for future needs and emergencies, and solving miscellaneous family economic problems. The detailed material which is being assembled under these subheadings will, it is expected, be related to important experiences of adults in which arithmetic of a simple kind can function.

The second large topic is "The Interpretation and Visualization of Quantitative Data," including the learning of concepts of commonly used quantitative units, the visualization of very large and very small numbers, the use and interpretation of tables, the construction and interpretation of graphs such as are commonly encountered in newspaper and magazine reading related to matters such as safety, peace, budgeting, social trends and progress, and the drawing of simple inferences from statistical facts of interest and concern to these pupils.

The third topic is "The Uses of Geometry," including in its subdivisions the observation and appreciation of the wealth of geometric forms around us, the measuring of lengths, angles, areas and volumes with instruments, the concept of the approximate nature of measurement, the solution of direct mensurational problems as applied to familiar objects in life, the solution of problems in indirect measurement including drawing to scale and simple numerical trigonometry, the use of instruments to construct geometric forms, the experimental discovery of common geometric relationships, and the concept of a dynamic view of geometry through the use of the idea of locus.

The fourth and last topic is "Algebra as a Tool of Thought." Here are included the use of the formula, in geometry and in arithmetic, the broadening of the number concept to include signed numerals, the extension of arithmetical operations to embrace literal numbers, the solution of simple verbal problems by means of linear

equations and pure quadratic equations.

It should not be assumed that the order in which the foregoing topics and their subdivisions have been read is the order in which they are to be taught. The matter of an optimum teaching sequence is still to be determined by the Committee.

A comparison of this very sketchy outline of the proposed course with what is generally termed academic mathematics reveals, first, the inclusion of a large unit of social arithmetic composed of elements related to life situations and experiences of responsible and socially minded adults; second, the complete absence of formal demonstrative geometry, which is, in my opinion, above the mental level of slow pupils; third, the elimination of the great bulk of manipulative algebra; fourth, the use of algebra as a tool, not for its own sake chiefly, but for solving useful problems arising in geometry and arithmetic.

In the interest of a clearer picture I should like to enlarge somewhat upon one or two of the sub-headings listed under "The Social Uses of Arithmetic."

Under "Recreational and Leisure Activities" are included the reading and interpreting of sports data of interest to boys and girls, arithmetical problems based upon authentic track and field events, hockey records, chess records, etc., puzzles for recreation and mental relaxation, magic squares, short cuts in multiplication, tests of divisibility of numbers, discussion of undesirable recreations such as "policy" games, and a study of the historical development of the Hindu-Arabic number system and its advantages over the Roman system.

"Health Activities" include the learning of quantitative standards of bodily functioning such as normal and abnormal bodily temperatures, pulse rate, respiration, blood pressure, height and weight, eye vision. Also included under "Health Activities" is the discussion of standards of balanced diet in terms of vitamins and calories, and the proper proportion of proteins, fats and carbohydrates. Tables of food values are employed. The treatment of this unit would be descriptive rather than manipulative. Another unit related to health is the learning of quantitative aspects of safety in automobile driving. Here the discussion could well center about tables giving stopping conditions for different speeds in good weather and bad, tabulations of deaths and injuries to pedestrians due to various causes such as jay walking, playing in the street, riding or "hitching."

Teachers need not, however, feel limited or circumscribed by any courses of study that have been proposed or are now in preparation. If it is agreed that mathematics should help these slow boys and girls better to understand their environment, quantitatively and spatially, one may turn to original sources for some material.

For example, what mathematics does the average person use in his daily life, excluding those activities directly related to his vocation? Here is a partial list submitted to me recently by teachers who had recorded instances of such use in their own lives over a period of one week:

Checking bills: milk, telephone, light, gas, laundry, groceries, restaurant meals.

Estimating the amount of cash to draw from the bank for a week's personal expenses.

Counting change received when making purchases.

Keeping ping-pong score.

Studying the possibility of giving up night school work and summer work and still being able to meet necessary expenses.

Examining the diagram of the floor plan of a theatre as printed on the program.

Comparing costs of various ways of spending the Easter vacation.

Centering a picture on a wall panel.

Centering a new rug on the floor.

Computing bank balance in checking account.

Checking fine imposed for overdue library books.

Purchase of table pad based upon measured size of table.

Purchase of automobile tires at given discount.

Rough scale drawing of a bungalow apartment.

Determining what time to get up in the morning in order to keep certain appointments and return within a specified time.

Estimating what time to start on a one-day trip to Atlantic City, which will include a three hour promenade on the boardwalk.

Checking upon AAA coupons received as 10% discount on sales price of gasoline.

Preparing income tax report.

Paying sales tax on purchase of automobile.

You will note that the mathematics involved in the items mentioned is very elementary and can be manipulated by slow pupils.

If one turns to the news and advertising columns of the daily press, he will find a surprisingly large number of items on a wide variety of topics of human, cultural and social interest in simple mathematical settings. I tried it recently and culled nearly forty different items from two newspapers over a four-day period. Lack of time prevents me from describing these items.

Enough has been said to indicate the possibilities with respect to assembling the content for what I believe to be an appropriate course for slow pupils. The next major question is, "What modifications of teaching methods are desirable for this group?"

In the first place it is essential that the pupil be led to realize why each topic is taught. He must be convinced, for example, that the making of personal budgets of money or time is a process of adjustment to inflexible material boundaries that builds self-respect, happiness and efficiency. If these ends are desired by the pupil, he will accept the study of budget making as satisfying a felt practical need. Similarly, the pupil should be led to see that the formula is a highly worth-while instrument which can be used to solve useful problems in social arithmetic or in mensurational geometry. It is not sufficient for the teacher merely to point out the usefulness or raison d'être of each topic in the syllabus at the time of its initial presentation. The pupil must be actively convinced, as a result of the teacher's persistent efforts in this direction, that the topic is well worth learning. Such conviction is important for its stimulating effect on interest and application.

Adequate time should be allowed for class discussion directed toward fixing basic concepts. For example, life insurance should be understood by the class to be a cooperative arrangement for the protection of person or property. The factors determining life insurance rates should be discussed, the different types of life insurance described, and their relative advantages and disadvantages noted. Similarly, taxation should be recognized as the means whereby the government, local State or Federal, is enabled to carry on its activities from which we all benefit. The meaning of the different kinds of taxes: sales tax, property tax, gasoline tax, water tax, automobile registration tax, State and Federal income tax, should be brought out. This descriptive treatment of taxes furnishes a valuable concrete setting and frame of reference which gives substance and meaning to the arithmetical exercises based upon this topic. References to these basic ideas should be frequently elicited in connection with the solution of the numerical exercises, in order that the latter may not degenerate into mere manipulation.

Every effort should be made to reduce the amount of formalism in exercises assigned for class work or homework. As far as possible, exercises should be selected or devised which make an appeal by virtue of their concrete setting. For example, a problem such as "Write your first-third marks in your major subjects and calculate your average mark" will be found to be more appealing than "Find the average of 75,80,70,50." An exercise such as "Find the area of the top of your desk" is preferred to "Find the area of a rectangle whose length is 12 and width is 5."

You are no doubt saying to yourselves that these points I have mentioned thus far under method are applicable to instruction of normal pupils too. Quite true,

except that the slower minds demand the fullest possible measure of motivation and concreteness of treatment in order that comprehension be effectively aided and interest maintained.

The teacher's procedures and devices should display considerable variety in order to combat the early flagging of attention characteristic of slow groups. Thus a period of active discussion might be followed by an interval of written exercise work at boards or in seats, or oral drill, or the use of visual aids, or measurement, or geometric construction. The activities here suggested have the further advantage of insuring multiple sense appeal, which induces comprehension and mastery.

The amount of drill planned for slow pupils should exceed the amount designed for normal pupils because the slow pupils need more practice to insure tolerable mastery. However, it is a serious and common mistake to overdo drill with these pupils, so that the class period becomes one monotonous grind of formal manipulations applied to lifeless material. Drill of this type is almost a complete waste of time; the probability of future recall of processes drilled upon under such conditions is exceedingly low. It is only as drill exercises are attached to meaningful and concrete situations that they help develop specific desired skills. In this connection too much criticism can not be leveled against formal drill in arithmetical operations for their own sake, completely divorced from social problems in which these operations function.

In view of the fact that the slow pupil has a slow moving mind, it is exceedingly important that sufficient time be allowed to him to think out the answers to the teacher's questions. Of course, it is assumed that the questions are suited to the ability level of the pupils. Accuracy should be stressed, guessing severely discouraged.

In this connection, may I state most emphatically that it is highly desirable that the development and review of a topic be carried on as far as possible through questions that stimulate them to think? Slow pupils *are* capable of thinking, although their thinking is on a comparatively low level. They can make easy generalizations and sense simple mathematical relationships. You need not tell them that the sum of the angles of a triangle is a straight angle. They will tell it to you after they have measured the angles with protractor, and cut them out and placed them together so as to form a straight angle.

I have stated that the motor skills of slow pupils approximate those of normal pupils. Inasmuch as measurements and constructions of figures and models make use of additional avenues of sense appeal, and provide, at the same time, concrete forms in which geometric relationships may be studied, it is recommended that much work of this type be done. The pupils enjoy construction work, and some of them show exceptional proficiency. However, it is important to emphasize that designs should be constructed less as ends in themselves than as means of better understanding and appreciating natural and artistic forms. Designs, posters and models should neither be elaborate nor time consuming. May I also state, in this connection, that the almost endless constructions of perpendicular bisectors of lines, angle bisectors, rectangles, squares, parallelograms, triangles, etc., tend to be excessively formal and uninspiring unless the constructions serve as means to an end, such as the experimental discovery of geometric relationships, or as an aid to locus work, or for the purpose of constructing scale drawings, etc.

A considerable difficulty encountered by slow pupils is reading for comprehension. The teacher must recognize this serious shortcoming and not expect too much in the way of text-book study.The recent inauguration of a remedial reading program for poor readers in New York

City high schools is a progressive and welcome innovation.

Human beings thrive on praise and encouragement, particularly when they feel that they are undertaking some difficult project or enterprise. The slow pupil fears that school work may constitute a difficult intellectual hurdle. He tends to lose confidence in himself at times, even when confronted by easy tasks. The teacher can help greatly by being generous with expressions of commendation such as "Fine!" "I knew you could do it!" etc. The pupil is impelled to do his best, and he does so with the feeling that the teacher is personally interested in him and wants him to succeed. Correlatively, the teacher should refrain from offending the sensibilities of the pupils by intemperate scolding or chiding when their accomplishment is below what is expected of them. The older children, conscious of the physical and emotional attributes of late adolescence become resentful or antagonistic; the younger ones may be completely discouraged and give up altogether.

The final point I desire to make with regard to method is that we should set standards of attainment commensurate with the mental level of the pupils and then insist upon maintaining these standards. Each pupil should be expected and required to work to the best of his ability. If content and method are adjusted to the degree of his intelligence and his mental characteristics, the slow pupil may be led to successful achievement. That this can be done is evidenced in schools where adjustments have been made. It is a serious mistake to assume that the slow pupil is necessarily lazy or disorderly or irresponsible. If these undesirable symptoms exist they usually spell educational maladjustment rather than natural delinquency.

A word now as to the qualities that a teacher of slow pupils should have. He should certainly possess an ample fund of patience, tolerance and sympathetic consideration for his charges. He should be able to step down to the level of the pupils in use of language and lucid presentation of ideas. He should have that reservoir of experience and knowledge that will enable him to vitalize such topics as social arithmetic and geometry in life. He should be a fine teacher in every sense of the word, one who takes a keen interest in these boys and girls and who looks upon assignments to teach such groups as opportunities for rich service.

It is obvious, I hope, from all that I have said, that the necessary adjustment of the educative process to slow pupils can not be effected unless these pupils are grouped into special classes. The basis of this segregation should be primarily the intelligence quotient, supplemented in doubtful cases by previous school achievement. The administrative machinery should be flexible enough, however, to permit those who display exceptional ability to be transferred to the academic course where they possibly belong.

The challenge of the slow pupil is urgent and formidable. Let us not aid the enemies of the teaching of mathematics by postponing longer a united cooperative effort to meet this challenge. We must recognize once and for all that square pegs can not be fitted into round holes. We must realize with finality that in the world of intellect all men are not created equal and that no teacher can make them equal. It must be evident to all of us that the solution of the problem lies in the direction of a thoroughgoing reorganization of objectives, content and method adjusted to the intelligence of slow pupils and suited to their personal and social needs. As I have stated before, in some schools tangible progress has already been made in the development of appropriate courses of study. In many other schools the spark has been kindled and serious work begun. In the presence of these hopeful signs we may all take heart and rejoice that mathematics will not soon give up its rightful place in the curricula of the secondary schools.

When Wishes Come True
A Mathematics Play*

IN TWO SCENES

By HANNAH A. PARKYN
Holland, Michigan

COSTUMES

SCENE I

Dick: A clown suit.
Father Mathematics: A long, flowing black robe decorated with various mathematical symbols cut from gilt paper.
Katherine: Coat and hat.
Ruth: (second entrance) An apron over an ordinary dress.
Other Characters: Ordinary clothes.

SCENE II

Divisor—Multiplier: A triangular paper hat, the front of which is blue with '3/2' on it in white and the back of which is white with '2/3' on it in blue. A mask worn on the back of the head. A dress the front of which has a blue skirt and white waist with '3/2' on it in blue and the back of which has a white skirt and blue waist with '2/3' on it in white.
First Traffic Cop: A silver star. A "club" made of a stick wound with blue paper and surmounted by a silver icosahedron. A red hat shaped like a frustum of an hexagonal pyramid, part of which is turned back for a brim. The brim is decorated with black mathematical symbols.
Second Traffic Cop: A gilt star. A "club" surmounted by a gilded dodecahedron. A hat like that worn by the First Traffic Cop.
$2X$, $3X$, X, $4X$, $5X$, 6, 7: Each has an appropriate cardboard placard. Each carries a square cardboard sign fastened to a piece of wood similar to the handle of a fan. On one side of the cardboard is a black positive sign and on the other side, a black negative sign. The one carried by $5X$ is made of two separate pieces of cardboard fastened together with paper clips. Flowing robes of various colors are attractive but not indispensable.
X^2: A crown. A flowing robe. A placard with x on it. A sign like those carried by the various X's.
Positive: Flowing robe. White placard with large green plus sign on it.
Negative: Flowing robe. White placard with large red minus sign on it.
Positive Parenthesis: Green flowing robe. One wears a large parenthesis cut from black cardboard on the right arm; the other wears one on the left arm.
Negative Parenthesis: Red flowing robes. Black parentheses like those worn by the positive parenthesis.
Herald: White trousers. A colored cape. A herald's hat if desired.
A, *B*, and *C*: Working clothes. Appropriate cardboard placards.
D: A cardboard placard with *D* on it. Clothes that give appearance of an old man.
Inspector, Collector, Messenger: Ordinary suits.

CHARACTERS

Marian Brown—A high school girl.
Bill Brown—Her brother, home from college.
Ruth Brown—Their sister.
Dick—A high school boy.
Father Mathematics—Friend of Humanity.
Bob—Friend of Marian.
Johnny Brown—The younger brother.
Katherine—Friend of Marian.
Members of Mathematics Family.
Divisor-Multiplier.
Two Algebra Traffic Cops.
X, $2X$, $3X$, $4X$, $5X$, 6, 7, X^2.
Positive, Negative.
Negative Parenthesis (2 people).
Positive Parenthesis (2 people)
Herald.
Inspector.
Messenger.
Collector.
A, *B*, *C*, and *D*.

SCENE I

Setting: The living room of the Brown family. The necessary articles of furni-

* Given for an assembly program by the Mathematics Department of Holland High School, Holland, Mich.

ture include a davenport, two easy chairs, a telephone and telephone stand, a radio, a floor lamp, and a small table or stand. The table contains a dictionary. A newspaper and a magazine or two are on the davenport. There is a doorway at each side of the stage and the curtains at the back are so arranged that a person can pull them apart and enter from the rear. When the curtain rises, Bill is sitting in one of the easy chairs, reading a newspaper, and Marian is on the davenport, studying.

MARIAN: Listen, Bill. If the sides of a quadrilateral are extended each its own length and the ends of the prolongations . . .

BILL: Say! Don't you know that this is my vacation?

MARIAN: Know it? How could I help it? Haven't I heard it enough times! If Dad wants you to put a shovel of coal in the furnace or Mom even suggests that you drive a nail, it's, "If you knew how hard we engineers have to work at Michigan . . . "

BILL: Well, we do work hard.

MARIAN: Is that so? At what?

(Ruth enters R. and looks among the newspapers and magazines on the davenport while Bill and Marian continue their conversation.)

BILL: Math., for one thing.

MARIAN: But you claim you like math.

BILL: But can't a fellow work hard doing what he likes?

RUTH: Where's the crossword puzzle page?

BILL:

(Searches for the page and hands it to her.)

Here, take it, but don't ask me about African antelopes or titles for legendary kings.

(Silence for a moment. Ruth sits down and immediately becomes absorbed in the crossword puzzle. She consults the dictionary occasionally.)

MARIAN: Say, what does the bisector of an angle of a triangle do to the opposite side?

BILL: Huh? Oh, it . . . it divides . . . say, Sis, that dress is pretty nifty.

MARIAN: Since it's taken you just two years to find it out, you can save your compliments. They won't do me any good anyway. Miss Reeverts* said we had to use supplements in this problem. If I had my way . . .

(Dick, dressed in a clown suit, enters L. turning a handspring or making a cartwheel.)

Just who do you think you are?

DICK:

(Bows.)

"A fool, sir, at a woman's service."

(Bows.)

BILL: A fool, 'tis evident, but which fair lady have you chosen thus to serve?

DICK: Miss Lindsley. It's just this way. She has to have somebody to take the fool's part in a Shakespearean play next year. She says that with practice I might do.

BILL: You might do all right, but just where did she get the idea that you need practice to play the part of a fool?

MARIAN: I'd much rather you'd play the part of a wise man and help me with this geometry problem. Just what did she say to do with these triangles?

DICK.

(Sits down long enough to glance at the problem. Rising.)

Why ask me?

(Bows.)

†"Oh, geometry, geometry!
A 'solid' study, you'll agree;
And what care I how 'plane' it be,
If it be not plain to me?
"And angles!—If your brain's 'acute,'
Your work is right and sure to suit.
But if you chance to be 'obtuse,'
To toil and weep, is not much use."

MARIAN: Come on, Dick. Be serious half a minute, won't you? Look here, how am I ever going to prove these arcs equal?

* Names of local people should be substituted in this and similar places.

† *The Mathematics Teacher*, December, 1926.

DICK: Why not ask Noah? He specialized in arks.

(Ruth looks up from the crossword puzzle.)

BILL:

(To Ruth.)

Just come to, Ruth? The time is a late winter afternoon. The place is the living-room of the Brown family on the planet Earth. The characters . . .

RUTH: Just this once, Bill, please, . . .

BILL: Oh, all right, but I warn you right now that I'm not going to look in half a dozen encyclopedias.

RUTH: What's an appropriate title for a knight named Kell?

(Dick saunters over to Ruth's chair, looks over her shoulder etc. during the next few speeches.)

BILL: An appropriate title for a knight? Let's see. You call a knight 'sir', don't you?

RUTH: Oh, I've got it. Sir Kell—circle. (Fills in the word.)

Now maybe we can get number three—a bus but not a taxi.

BILL:

(Ponders, then his face lights up.)

How about a rhombus? It's not exactly a taxi but how it taxes our sweet sister's brain.

DICK:

(To Bill)

Well, I bet you don't know this one even if you have taken calculus. What's a tall coffee pot in use?

MARIAN:

(Laying down her book.)

Give it to me and I'll see that it's in use. I'm starved.

BILL: A tall coffee pot in use? Who sprang that one? It sounds like one of Mr. Leddick's brain teasers.

DICK: Yes, he stopped right in the middle of a sentence about Pythagoras to test Harry Irvin's fitness for domesticity.

BILL: Thanks for the hint—hypotenuse, of course, high pot in use.

DICK: What hint?

MARIAN: Anybody that knows anything about Pythagoras . . .

DICK: But I don't. That's not a fool's job.

RUTH: Next I suppose you'll want a name for an angle that won't give up.

MARIAN: Well, I give up.

RUTH: Triangle.

DICK: Speaking of triangles, how is the Overbeek-Notier-Bosworth triangle progressing?

MARIAN: Irene seems almost equally inclined to both. One of them needs to invent a longer line.

BILL: Yes, a girl always objects if a fellow takes the shortest distance between two points when he's taking her home from the show.

RUTH: Well, there are plenty of triangles. Renetta Shackson, Bud Hertz, and Ted Essenburg form another.

DICK: Art Elferdink acts as if he were trying to make a quadrilateral out of it.

MARIAN: And the way the sophomore girls talk, you'd think La Verne Scheerhorn was the center of a polygon with an infinite number of sides.

(The telephone rings.)

Hello,—Yes, Katherine?—Dick is here. —No, he hasn't. He's been too busy getting rid of his surplus intelligence.—Would Bob and I like to go to Grand Rapids to the South game with you and Dick? I'd just love to. Has Dick asked Bob yet?—Then everything is all settled.—6:15, yes, that's O.K.

RUTH:

(Rising.)

Now, Marian, you know very well that Dad said you couldn't go to another thing until you got that geometry made up.

MARIAN: Well, I've got just nine more problems to do.

RUTH: Nine more to do! That means you've done one in the last four hours.

MARIAN: And I promised Mother I'd make a cake. Make it for me, won't you?

(Ruth frowns.)

I'll let you wear that new blouse of mine.

RUTH:
(Reluctantly.)
Well . . .
(Exits R.)

MARIAN:
(To Dick.)
You'd better run along now. I've got to work and you've got to fool.

DICK:
(Rises and bows elaborately.)
The fool ever does the queen's bidding.
(Bows.)
*"If you plant cube roots in the ground,
what grows?
And where has polygon, who knows?
And where does an angle get its degree?
And is it B. A. or Ph. D?"

MARIAN: Aren't you ever going? I'm sick of your brand of humor. You'll be singing the multiplication tables next.

DICK:
(Exits L. but returns to say tormentingly.)
*"Could the powers of x make a treaty, pray?
And does heat expand a binomial, say?"

(Marian hurls a book after him. He ducks, and disappears. She resumes her studying. Father Mathematics enters very quietly at the rear. Neither Bill nor Marian notices him.)

MARIAN: How I hate geometry! Yes, and arithmetic and algebra, too. I wish I never had to see or hear or be benefited by another bit of mathematics.
(Bill rises and starts to go.)
Why, where are you going, Bill?

BILL: Out of your sight before you banish me, too.
(Exits L.)

FATHER MATH:
(After a pause during which Marian studies.)
Pardon me for not ringing the bell but you see I came in by way of the fourth dimension. And, of course, since you are merely a three dimensional creature you couldn't see me until I chose to reveal myself.

MARIAN:
(Who has risen and appears startled.)
Who are you and why did you choose to reveal yourself at all?

FATHER MATH: I'm Father Mathematics.

MARIAN:
(Aside.)
As if I didn't have troubles enough already!

FATHER MATH: And I came to grant your wish.

MARIAN: My wish? To banish mathematics?

FATHER MATH: Yes, but with a few restrictions.

MARIAN: I suppose that I'll have to do those problems first.

FATHER MATH: No, indeed. You needn't do them until you really want to. I'll withdraw mathematics if you'll consent to my doing it gradually and . . .

MARIAN: Yes, go on.

FATHER MATH: And if you'll let me so control the situation that it won't be necessary for any one to die for lack of mathematics.

MARIAN: Imagine anyone's dying for lack of mathematics! It wouldn't affect even Miss Reeverts that much. And I don't have to do those problems?

FATHER MATH: That is the agreement. Do you consent?

MARIAN: Who wouldn't? Be sure to take away those awful numerical computations, ratio and proportion, locus of points,—how I hate that stuff—and
(Looks in book.)
regular polygons. We're just ready to start them.

FATHER MATH.
(Sits.)
That's a big order but I promise.
(Looks at watch.)
In five minutes I shall begin withdrawing the many branches of my family from duty.
(The doorbell rings and Marian goes to

* *The Mathematics Teacher*, Dec. 1926.

the door just as Bob enters L. with his violin case.)

MARIAN: Hello, Bob. I see you brought your violin. You'll have to help me celebrate the fall of mathematics. Come, play your gayest tune.

BOB:

(Bewildered.)

But . . .

MARIAN: But I'm forgetting my manners. You've never met Father Mathematics before, have you?

(They exchange greetings.)

He's promised to withdraw mathematics from the world—Oh, not quite all of it and not all at once—And I needn't do those pesky problems.

FATHER MATH: Quite true.

(Glances at watch.)

But let's talk of other things. I'd like to hear that violin. Won't you play it for us now?

BOB: I'd be glad to.

(He takes violin from case while Marian and Father Mathematics seat themselves. As he plays they listen with evident enjoyment for about three minutes. Then Father Mathematics looks at his watch, rises reluctantly, and touches the violin lightly. Immediately discords being. Bob looks puzzled and tries to play as usual.)

MARIAN: Is something broken?

BOB: I don't know what's the matter.

FATHER MATH: Perhaps I can explain. I agreed to take away ratios. Now musical harmony depends upon the ratio of the frequencies of the vibrations set up. You don't want ratios. So I don't see how you can have harmony.

MARIAN:

(To Bob.)

But you don't have to know all about ratios in order to play, do you?

BOB: No, but I guess they are there just the same.

FATHER MATH: Aren't there—now, you mean.

MARIAN: Well, I didn't know that math. had anything to do with beautiful music

(Bob puts away his violin and sits down.)

FATHER MATH: It has much to do with all beauty.

RUTH:

(Enters R. wearing an apron and carrying a mixing bowl and egg beater.)

You know those flowers we bought as a surprise for mother?

MARIAN: Yes, what's happened to *them*?

RUTH: They were just as fresh as could be when I started making the cake and when I looked just now the petals were all falling off.

MARIAN: Some more of your ratios, Father Mathematics?

FATHER MATH: No, this time it's regular polygons. The tips of the petals form one. Of course you know that most artificial designs are based on geometry, but much of the beauty in nature depends on my laws, too.

RUTH: What queer talk! Everything's queer around here. Every measuring cup in the place breaks when you look at it. By the way, did you order those groceries Mother wanted?

MARIAN: No, I forgot. I'll telephone right away.

(Ruth exits R.)

BOB: Don't be too sure of that.

MARIAN: Why, what do you mean?

(Attempts unsuccessfully to telephone.)

I guess you're right. Turn on the light, won't you?

(Bob rises, goes to floor lamp and pulls the cord without results.)

No light either.

BOB:

(Turns radio dials.)

And of course the radio won't work. There's a lot of mathematics connected with electricity, you know. I've seen all sorts of equations in a book about it.

(Sits down again.)

JOHNNY:

(Enters L.)

Hi, everybody.

MARIAN: You're just in time, Johnny.

Won't you go to the store to get some groceries? The phone's out of order.

JOHNNY: Sure, I don't mind. What do you want?

MARIAN: Eggs, for one thing.

JOHNNY: How many?

MARIAN: Well . . . er . . . oh . . . some. I can't tell you how many. And sugar . . . some of that, too, . . . and quite a lot of potatoes and a few rolls and . . . well, never mind the rest.

JOHNNY: Say, what's the matter with you? Mom always tells a fellow how much to get.

MARIAN: Well, there used to be numbers but there aren't any now.

JOHNNY: Oh! Boy! No more multiplication tables! Well, so long.

(Takes out a large candy bar and unwraps it very deliberately.)

MARIAN: Just a bite, please. I'm famished.

JOHNNY: Can't do it, Sis. If there aren't any numbers, there's no such thing as division.

(Takes a huge bite and grins mischievously at Marian as he exits R.)

RUTH:

(Enters R.)

What's wrong with the recipe books? I know one thing; there'll be no cake in this house for supper to-night.

(Exits R.)

KATHERINE:

(Enters L. dressed for the street, and sits down in the nearest chair as she talks.)

Just came in to tell you that the Grand Rapids trip is all off.

MARIAN: What's the matter now, Katherine?

KATHERINE: Something's wrong with the headlights on the car and not a garage in town can fix them. All the mechanics would do was to jabber about mathematics.

BOB: Are you sure it's just the headlights? Of course the reflecting surface is the locus of points which . . .

MARIAN:

(Wrathfully.)

So that's some more of your work, Father Mathematics. I think you're mean.

(Bursts into tears.)

FATHER MATH: Don't feel too badly. There won't be any game anyway.

ALL: No game!

BOB: That's right. They can't keep time or count score.

KATHERINE: But couldn't they just play anyway?

FATHER MATH: No, indeed. The path a basketball makes comes under locus of points too.

MARIAN: How I hate the very word locus! It always makes me think of the ten plagues of Egypt.

BOB: Well, isn't it time we did something? The banks will all be closed; bridges will collapse; lots of factories won't be able to run. Father Mathematics, what are your terms?

MARIAN: Yes, what are they? I'll promise anything. I'll even work those problems.

FATHER MATH: We might have a truce to talk things over.

MARIAN: Well, I'm the one to blame. So the rest of you needn't stay unless you like.

(Bob and Katherine exit R.)

FATHER MATH: First, I'm going to ask you to take a trip with me. Travelling is really very easy when you ride in the Fourth Dimension Dirigible.

CURTAIN

SCENE II

Setting: The Land of Mathematics. The stage is bare. As the curtain rises, Marian and Father Mathematics are walking slowly across the stage from the right.

MARIAN: Perhaps the trouble has really been that I don't know enough about the mathematics family.

FATHER MATH: Wouldn't you like to meet some of them?

(Divisor-multiplier enters from the rear of the stage and walks backwards.)

I see one of my daughters coming now.

Miss Brown, may I present Divisor 2/3?

(Divisor bows and shakes hands backwards.)

Some people think her a trifle awkward but that's because they look at her the wrong way. Really we couldn't accomplish much without her.

(Divisor turns around.)

And here is Multiplier 3/2.

MARIAN: But I don't understand. I thought you said her name was Divisor 2/3.

FATHER MATH: So I did and so it is. Don't you see that she's two people in one? Sometimes her mask frightens the children but after they once see that she's really the agreeable Miss Multiplier, they don't mind her at all.

(Divisor-multiplier exits slowly R.)

ALGEBRA TRAFFIC COPS:

(Enter briskly L. and bow.)

We are the Algebra traffic cops.

(Bow.)

(Sing.)

*In the running down of signs,
We are the winners every time.
We show negative the place to go.
We run across a careless sign
And then we jerk him roughly out of line.
'Tis the duty of a cop.
If they will follow our direction
And will act with circumspection
Remembering very well the laws by Math. laid down,
Then all in harmony may dwell
With no one in a stuffy cell, you know,
It is the duty of a cop.

(Bow.)

And we are the algebra traffic cops.

(With a flourish they take up positions on opposite sides of the stage. X, $3X$, $2X$, X^2, $5X$, and $4X$ enter R. 7, 6, Positive, Negative, and all the Parentheses enter L.)

* Parody on "The Duty of a Cop" from the operetta, "*Pickles*," H. T. Fitzsimons Co., Music Publishers, Chicago, Illinois.

FATHER MATH: You recognize my numerical children, don't you?

(7 and 6 nod pleasantly.)

And these are Parentheses—they always come in pairs, you know—

(They bow.)

And these are some of the various X's. They all do double duty for each one carries two signs.

(X and $5X$ rush up to Marian and display signs.)

MARIAN: How do you do, Positive X and Negative $5X$. I remember you.

(To Father Mathematics.)

I've met all of them before, I think, except the tall one with a crown. Just who is he?

FATHER MATH: Oh, I supposed you knew. That's X raised to a power. The minute he puts on his crown—we mathematicians call it an exponent—well, you should see him expand.

HERALD:

(Enters R. Blowing a trumpet.)

Hear ye! Hear ye! The algebra inspector is on his way.

TRAFFIC COPS: Parentheses formation.

(The other characters step briskly to positions to form

$$+x+(x^2-2x+6)-(4x-7+5x)-3x.$$

The parentheses stand sideways in such a way that the arm with the parenthesis on it is toward the audience. $4X$, 7, and $5X$ take a sharp look at Negative before they step within the parentheses and reverse the signs which they carry. The traffic cops see that the line formed is straight, that the signs are held correctly, etc. Then the inspector enters R.)

FIRST TRAFFIC COP: Ready, Positive Parenthesis?

POSITIVE PARENTHESIS: Ready, your honor.

SECOND TRAFFIC COP: Ready, Negative Parenthesis?

NEGATIVE PARENTHESIS: Yes, your honor.

(The herald plays and X steps forward. There is a pause. He plays again while Positive and Positive Parenthesis step to

the back of the stage and X^2, $2X$, and 6 step forward. There is a pause. The herald plays a sharp warning note, pauses slightly, and plays again. Negative and Negative Parentheses step back. $4X$ and 7 reverse their signs and they and $5X$ step forward. $5X$ appears to be day-dreaming. There is a pause. The herald plays again while $3X$ steps forward.)

SECOND TRAFFIC COP:

(Grasps $5X$.)

$5X$, I arrest you in the name of the law.

$5X$: What law?

TRAFFIC COP: The law of signs. You were in the *Negative* parenthesis, you know.

(Takes him to Father Mathematics.)

FATHER MATH: Guilty or not guilty?

$5X$: Guilty, your honor. Please pass sentence at once.

FATHER MATH: $5X$, since you fail to appreciate the privilege of being able to change your sign, I sentence you to forfeit your negative sign. From now until June 17, you may have only one sign, and you must take it with you wherever you go.

($5X$ hands over his negative sign.)

INSPECTOR: I sustain the verdict.

(The X's and 7 step to the back of the stage. The herald, playing as he goes, and the inspector exit L. A messenger enters L. and hands Father Math. a telegram.)

FATHER MATH: There's trouble at the Electrical Research Bureau in Washington, and they want help. I fancy that what they need is a few parentheses, the powers of X, and perhaps the numerical relations.

(Looking at watch.)

There's a train at 8:15, you'd better take that.

(The Parentheses, 6, 7, and X^2 exit L. Collector enters R. with two butterfly nets.)

Oh, here comes the collector from the mathematics museum. Something for you, sir?

COLLECTOR: May I have your stray X's today?

FATHER MATH: Yes, though I hate to part with them.

$2X$:

(As $2X$, $3X$, and $5X$ display their respective signs.)

None of us are stray.

$3X$:

We just balance one another.

COLLECTOR: Then I'll not detain you.

($2X$, $3X$, and $5X$ exit R.)

So you're left alone, $4X$ and X. Then you may come with me.

(Throws a butterfly net over the head of each and they exit R. The traffic cops walk off L. singing the first few lines of their song.)

FATHER MATH: Perhaps you'd like to meet someone who could tell you about A, B, and C. The algebra books are full of their adventures.

MARIAN: Adventures? Oh, you mean those awful problems in which A, B, and C pump water into cisterns, run races, chop wood, and invest their earnings?

(D enters L.)

FATHER MATH: Yes.

(To D who is walking across the stage.) Oh, Mr. D, won't you tell my friend something about A, B, and C? You've known them a long time, haven't you?

D: †Ever since they were little fellows in brackets. We used to call them John, William, and Henry in those days. Haven't you read about their marble tournaments? Sure you have. A always won four times as many marbles as C. How he used to boss poor little C around! The college sheiks, they say, have called in three foreign experts, Alpha, Beta, and Gamma, but A, B, and C still do piles of work for common folks. Isn't this C coming now?

C:

(Enters R. coughing.)

† This and the following speeches follow quite closely the ideas and the conversation in "A, B, and C" from *"Literary Lapses"* by Stephen Leacock.

Hello, *D*, how are you these days and what are you doing?

D: I scratch around in my garden a little, raise a common denominator or two and grow a bit of logarithm. In harvest season I extract a few roots. Occasionally, too, Durrell and Arnold call me in for demonstration purposes, but I'm getting too old for hard labor.

(Exits R.)

B:

(Enters L.)

Well, *A* just beat me in a rowing match. I had a start of forty-five strokes but he managed to overtake me at the second bend. And to-morrow all Mr. Carter's classes have to figure out his rate per minute.

A:

(Enters L.)

Say, fellows, Mr. Leddick has found three cisterns in his back yard, and he says we can have until to-morrow night to pump water into them. I bet I can beat you both. Your cistern leaks a little, I think, *C*.

B: Why don't you take the leaky one for a change? Don't you know that *C*'s all in?

(*C* coughs at intervals.)

A: Then Mr. Carter wants us to come over to mix up some salt solutions.

B: And I suppose that *C* and I will have to carry pails and pails of water while you'll take the one that needs three and eight-nineteenths pounds of salt.

A: While we're waiting for the Leddicks to finish supper, we might chop wood for Mr. Riemersma.

B: Look here, *A*, *C* isn't fit to chop wood.

C: I might try but I'm afraid I won't be able to work even a fourth as fast as *A*.

B: I'm going to fetch a doctor. He's in bad shape.

A: You've no money to fetch a doctor.

B: I'll reduce him to his lowest terms. That'll fetch him.

(Exits L.)

A:

(After a pause.)

If I walk twice as fast as *B*, I can beat him there yet.

(Exits hurriedly, L.)

FATHER MATH:

(Laying his hand on *C*'s shoulder.)

Don't worry, *C*, we won't let anything happen to you, you're too valuable a man to lose.

MARIAN: I think so too. We'll just have to keep the whole family.

(All characters appear on stage. The traffic cops see that they arrange themselves in orderly fashion and direct them in singing.)

(Tune: "We're loyal to You, Illinois.")

We're loyal to you, Father Math.
We're working with you, Father Math.
We're sure we can do
Every problem clear through,
For we know we need you, Father Math.
So bring on your work, Father Math.
We'll never say shirk, Father Math.
Our task is the finest ever,
Lead us in our endeavor,
Lead on for us, Father Math.

CURTAIN

Back Numbers Wanted!

THE MATHEMATICS TEACHER will pay 50¢ each for the following back numbers:

1. Vol. I, No. 1, September 1908.
2. Vol. II, No. 3, March 1910.
3. Vol. II, No. 4, June 1910.
4. Vol. III, No. 1, September 1910.
5. Vol. III, No. 2, December 1910.
6. Vol. III, No. 3, March 1911.

A Child's Concept of Numbers*

By GEORGE H. WHITEAKER
Brighton Junior High School, Brighton, Colorado

NO ADULT will ever be privileged to see or feel a six-year-old's concept of "seven" or "eleven" or to know just when it is adequate, but with the passing of the overworked bond-theory of learning it becomes very important that we know something of the pupils' concepts at every stage of instruction. So long as drill was the principal or only tool of learning it mattered little what the pupil knew or thought about the numbers he used. It was only important that he memorize them in the right places in the lines that he learned to speak. Now, however, meaning is replacing drill as a tool of learning, and meaning for a mathematical operation can only be based upon a workable concept of every factor involved in the operation. If mathematics teachers are to keep step with the new psychology of learning, it is necessary that they apply the criterion of meaning to every item in the mathematics course from the first grade up. Learning can be expressed only in terms of preestablished concepts. For example, if the child's only concept of numbers is one of serial relationships, it should cause no surprise when he counts to find a sum. If his only concept of a fraction is a number of equal parts of a whole, he will have trouble with three-fourths of eight apples.

In the matter of learning what concepts children hold, the field has only been scratched. One finds practically nothing in the literature that touches on concepts of number among children above the first grade. Here, the work of preparing concepts was started long ago. Grube[1] recognized long ago that rote counting might miss the point and outlined a long and tedious program of practice in counting objects. In 1927 Morton[2] warned against the acceptance of rote counting as evidence of ability to count objects. He reports a conversation between a little girl and her father to illustrate the point. The little girl is looking at a picture of four soldiers and counting them for her father. At the first try she counts *six* and at the next *seven*. It should cause no surprise that the child who has learned to count by rote should pronounce the number names without reference to the figures in the picture. To her *three* is the stimulus for *four*, and *four* is the stimulus for *five*. She has recited them thus many times just for the pleasure of vocalizing and to assure herself that she has not forgotten them. The four soldiers are only the stimulus to begin counting, for she has learned that counting is an appropriate response in the presence of any group of similar objects. We do not agree with Morton that rote counting is useless, but rather with Wheat,[3] who in a very recent book, gives the opinion that it is a necessary and useful prelude to the counting of objects. We must, however, go farther than either of these writers and point out that counting, even of objects, is no indication that the pupil knows the numbers he uses or what he knows about the ones he knows.

Josephine McLatchy[4] found that only two children in thirty-five could count no farther farther than five on the first day of school. She found that, on the average, they knew several combinations especially if the numbers were represented by groups

* Reprinted from Bulletin No. 1, Vol. III of the Mathematics Section of the Eastern Division of The Colorado Education Association.

[1] Seeley, Levi, *Elements of the Grube Method*, Kellog and Company, 1893.

[2] Morton, M. L., *Teaching Arithmetic in the Primary Grades*, D. C. Heath and Co., 1937.

[3] Wheat, Harry Grove, *The Psychology and Teaching of Arithmetic*, D. C. Heath and Company, 1937.

[4] McLatchy, Josephine and Buckingham, B. R., "The number abilities of children when they enter grade one." 29th Yearbook of the National Society for the Study of Education, p. 460. Also, "Number abilities of first-grade children." Childhood Education, 1934, 11: 34.

of objects exposed for a short time. She mentions being struck by the frequency with which children announced, "I know *two* and *two*." These findings are important. They have been corroborated by other investigations. The trouble is that they are so easily misinterpreted. Children are excellent parrots. They remember without effort, all sorts of meaningless statements and phrases. It would be surprising if some of the unintelligible things that are always pouring through their minds did not refer to numbers.

The commonest mistake is to assume that since the child appears to already know several numbers and number facts he can easily and quickly learn a lot more. These findings have been used to dispute the findings of the famous Committee of Seven.[5] The mistake arises from failure to understand that additional conceptions must come slowly, one by one, and are not readily transmittable. The child who glibly recites a half-dozen addition combinations during his first schoolroom interview, may still be struggling with combinations four or five years later.

The things that he has learned in the meantime are not to be measured in terms of the forty-five combinations. They are only a small part of his education. He has also had to learn many other things about every number within his acquaintance. Knowing the proper place to say *seven* in a list of nonsense syllables made him look precocious on his first day in school. Since then he has learned that *seven* was the age of his friends in the second grade, that a *seven* at the top of a page means the seventh page from the beginning of a book, that *seven* on a clock face means seven o'clock in the morning or evening, that *seven* on a ruler means seven inches from the end or seven inches in length for an object that reaches just that far from the end of the ruler, and that Saturday is the seventh day of the week. In the meantime, he has learned that *seven* is at one time a *two* and a *five*, at another time a *three* and a *four*, and that it is *three* less than *ten*. He has learned that two *sevens* are fourteen and that *seven* is a touchdown and the extra point. He has learned that there is a *seven* in every decade, but that those numbers are not multiples of *seven*. He has been intrigued by the ideas of seven *tens* and seven *hundreds*. If he has not learned that seven and eight are fifteen, that is only one less important item out of many.

Perhaps the least important concept of a number is that of being one in a series. Numbers were most probably first used as the names of groups[5] or as adjectives describing groups and without any serial relation at all. As soon, however, as consideration went to groups too large to grasp as a unit, systems of tallying were invented. These usually consisted of matching the objects to be tallied[6] with the fingers, and so the names of the small numbers became the names of the fingers or gestures used in tallying. No doubt the child first sees *two* as a pair and *three* as a group of three and would develop his conceptions to include a group of four, and when properly arranged, of five; but before he gets a chance to understand them, they are lost in the serial relationships forced upon him by benevolent elders.

The new psychology teaches us that it is a waste of time and a needless drudgery to teach the child things that he does not understand, or for which he is not prepared by a group of basic conceptions. Some very recent and very fine textbooks written around that theory of learning omit all the customary drill from the first grade and most of it from the second. By old standards they are merely readers; but for the teachers who have the courage to lay away the flash cards, forget the monthly tests, and work for the less tangible ends of concepts and meanings, better results and attitudes toward number work and school in general will be had.

[5] Washburne, C. W. et al., "The grade placement of Arithmetic." 29th Yearbook of the National Society for the Study of Education. p. 641–670.

[6] Judd, C. H., *Psychological Analysis of the Fundamentals of Arithmetic*, University of Chicago, 1927.

◆ THE ART OF TEACHING ◆

Teaching the Law of Signs in Multiplication

By ELBERT FULKERSON
Principal, Training High School, Carterville, Illinois

ONE WELL-KNOWN first year algebra text of a recent copyright, in introducing the multiplication of signed numbers, makes this statement:

"Really to understand the reason for the rule for multiplying signed numbers is quite difficult, and we shall state it here without giving any reason, and gradually you will learn to understand why this rule is what it is."

Immediately following this statement appears an italicized paragraph giving the rule, which precedes a group of exercises to be solved thereby. In the light of some of the generally accepted objectives to be attained in the teaching of secondary mathematics, can an author be justified in giving such a deadening introduction to a process relatively easy to rationalize?

THE OVER-LOOKED OBJECTIVE

One of the main purposes of mathematics is to provide the student with those mathematical abilities, meanings, and concepts which will assist him later in making satisfactory adjustments to the various problems of life. Algebra for the most part represents abstract processes where a certain amount of learning of the memoriter type is both desirable and essential. Some teachers, however, rely so much upon this type of learning that they are apparently satisfied when their students have been able to memorize and apply certain principles and rules regardless of how little meaning, significance, and thought-provoking stimulus may accompany the mechanical repetitions and blind manipulations involved in the processes.

To subordinate the rationalization of processes in mathematical instruction to the mere memorization of rules and the mechanical application of principles is indeed a sure way to dampen the enthusiasm of students and kill their initiative in this and similar situations where habits of straight-thinking ought to be used for the solution of problems, the avoidance of errors, and the discernment of truth—habits ordinarily regarded as fundamentally essential to any individual who would make satisfactory adjustments to modern society and live happily and successfully with his fellow human beings.

Of course the rationalization of processes for a ninth grade algebra class will require some additional effort, patience, and perhaps versatility on the part of the teacher; but the satisfaction experienced through added enthusiasm, quickened responses, and increased achievements on the part of the students, will liberally repay for the extra time and work, to say nothing of the lasting outcomes more likely to result from this type of instruction. If such is the case, then the mathematics teacher who would present his subject with meaning and lasting significance for the students, will take advantage of every available opportunity to rationalize processes which are not too difficult to be brought within the comprehension of his particular class.

No little experience in teaching first year algebra, and in supervising critics and student teachers in the subject, has convinced the writer that the reason for the law of signs in the multiplication of signed numbers can be made clear to the

average class if the approach is properly planned. Various procedures have been worked out for such a presentation, but the one given here seems to get the best results with the average class in first year algebra.

Developing the Process

The method here explained assumes that the students have already been familiarized with certain basic algebraic concepts such as the meaning of a term, numerical coefficient, literal coefficient, and signed numbers, and that they have at least a fair knowledge of addition and subtraction, including the removal of symbols of aggregation preceded by either a plus or minus sign.

The recitation should begin by calling upon students to review these concepts and principles with the class. Then some student will be asked to name the fundamental processes thus far studied in the course. Another student will be called upon to mention the next fundamental process ordinarily following addition and subtraction. Obviously he will say it is multiplication. At this point a simple numerical problem in multiplication will be written on the board, and some student will be asked to solve it, after which the multiplicand, multiplier, and product will be identified and explained by the members of the class. At this point the teacher will call the attention of the students to the fact that thus far they have learned to add and subtract signed numbers, and now they are ready to take up the multiplication of such numbers.

Now the teacher will lead the students to see that multiplication of signed numbers can involve only four possible combinations of signs; namely, multiplicand and multiplier both positive, multiplicand negative and multiplier positive, multiplicand positive and multiplier negative, or both multiplicand and multiplier negative. Then simple examples such as the following will be written on the board to illustrate these combinations and to be used in the explanations:

Multiply:	(1)	(2)	(3)	(4)
	5	−5	5	−5
	3	3	−3	−3

In considering the first example, the class is asked if there is any other way by which the value of 3 times 5 may be found other than multiplying 5 by 3. The response will readily come that the result may be obtained by using 5 as an addend three times. Then the example will be rewritten in the form $5+5+5$, and a member of the class will be required to find the result by an application of the rule for the addition of signed numbers. No student will have any difficulty in seeing that the sum is 15, and that, therefore, the product in the first example is likewise 15, which then is placed under the example as the result of the first multiplication.

The second example will be treated much the same as the first, but here 3 times -5 is written as an additional problem in which -5 is used as the addend three times, or $(-5)+(-5)+(-5)$. Again someone will be asked to obtain the result to this example by use of the addition rule. Here, also, the students will readily ascertain that the sum is -15, and that the product in the second example must likewise be -15, which is now written as the answer to the second problem.

In order for the class to understand the explanation connected with the third example, the commutative law of multiplication is developed by asking which is the more, 5 times 3, or 3 times 5. Obviously the answer will be that the two products are the same. Then the statement is generalized by asking what effect is produced upon the product by interchanging the multiplicand and multiplier. Again the class will conclude that the product will remain unchanged. From this it will follow that -3 times 5 is the same as 5 times -3, which may be written in

the form of an addition problem with -3 used five times as an addend, or $(-3)+(-3)+(-3)+(-3)+(-3)$. Here, as in the second example, by applying the addition rule the sum will be -15, which is also the product. This result is then written under the third example.

The rationalization of the process involved in the fourth example is a somewhat more difficult procedure than found in the previous three, but the following plan of attack will obviate most of the trouble. Such an example as $3x$ is written on the board and a student is called upon to identify the numerical and literal coefficient. Then x is next used in the same manner. Here the student will quickly respond that the numerical coefficient is 1. After this $-x$ is considered and the answer will then be that the numerical coefficient is -1. Following this the teacher will ask the class to express $-3x$ as an addition problem where -1 is the coefficient of x Someone will make the statement that $-3x$ is equal to $(-1x)+(-1x)+(-1x)$. After this conclusion, the class will be asked to consider the possibility of expressing $(-3)(-5)$ in a similar manner. At first there may be some hesitancy, but if so, the difficulty may be removed if the teacher will point out that the -5 may be thought of as the x in the previous illustration. Then $(-3)(-5)$ would be written as $(-1)(-5)+(-1)(-5)+(-1)(-5)$.

To clarify the procedure it is well to group the addends in the last example in the form of a problem in vertical addition thus:

(a)		(b)		(c)		(d)
$(-1)(-5)$	equals	$-1(-5)$	equals	$-(-5)$	equals	5
$(-1)(-5)$	"	$-1(-5)$	"	$-(-5)$	"	5
$(-1)(-5)$	"	$-1(-5)$	"	$-(-5)$	"	5
$(-3)(-5)$	"	$-3(-5)$	"	$-3(-5)$	"	15

The teacher should write column (a) on the board, and then have the students derive columns (b), (c), and (d) from the first column. They will have no difficulty in seeing how column (d) follows from (c), since they have previously learned the principle of removing symbols of aggregation preceded by a minus sign. From this they will discover that -3 times -5 produces 15. This result is written as the answer to the fourth example.

The final step is to have the students examine the answers to each of the four examples with a view to making a generalization on the multiplication of signed numbers. The teacher will be surprised to see what a relatively large number of students in the class will be able to do this at the close of the presentation. Just recently the writer made a similar presentation to his high school algebra class, and then asked the class to formulate a rule from the discussion. The first student on whom he called gave this generalization: "In multiplying algebraic quantities if the signs are alike the answer will be positive, and if the signs are unlike the answer will be negative." Such a clear-cut statement of an algebraic principle on the part of a student who had just participated in the rationalization procedure would seem to warrant the conclusion that it is not a difficult problem for the average first-year algebra student to understand the rule for the multiplication of signed numbers, provided the teacher will use a little time and effort in making the proper presentation of the materials at hand.

THE NATIONAL COUNCIL of Teachers of Mathematics Nineteenth Annual Meeting will be held February 24 and 25, in Cleveland, Ohio. Headquarters—Carter Hotel. Reservations direct to hotel. Rates per room, *single or double occupancy;*

Twin beds $6, $7, and $8
Double beds $4, $5, and $6.

A. BROWN MILLER, *Chairman, Committee on Local Arrangements*

Program, National Council of Teachers of Mathematics

Twentieth Annual Meeting

Carter Hotel, Cleveland, Ohio, February 24, 25, 1939

FRIDAY, FEBRUARY 24, 9 A.M.
North Georgian Room

Board of Directors Meeting

FRIDAY, FEBRUARY 24, 2–4 P.M.
Directoire Room

I. Training Teachers of Secondary School Mathematics

Presiding: John P. Everett, Western State Teachers College, Kalamazoo, Michigan

1. The General and Subject Matter Preparation of High School Teachers. L. H. Whitcraft, Ball State Teachers College, Muncie, Indiana
2. Changes in the Mathematics Program Suggested by Recent Trends. Lee E. Boyer, State Teachers College, Millersville, Pennsylvania
3. Present Day Demands in the Study of Mathematics. W. W. Rankin, Duke University, Durham, North Carolina

FRIDAY, FEBRUARY 24, 2–4 P.M.
Grand Ballroom, North End

II. Mathematics and Its Uses

Presiding: E. R. Breslich, University of Chicago

1. The Mathematics of Most Value to Pupils of Science. Ira Davis, University of Wisconsin
2. Mathematics in the Practical Affairs of Life. W. S. Schlauch, New York University
3. Mathematics in a Science High School. Julius H. Hlavaty, High School of Science, New York City

FRIDAY, FEBRUARY 24, 4–5 P.M.
Grand Ballroom, South End

III. Tea and a get-acquainted and see-exhibits hour.

IV. New Phases of Mathematics Teaching

Presiding: H. C. Christofferson, President N.C.T.M., Miami University, Oxford, Ohio

FRIDAY EVENING, FEBRUARY 24, 8–10 P.M.
Grand Ballroom

1. Address of Welcome. Charles H. Lake, Superintendent of Cleveland Schools
2. Features of Progressive Education Applicable to Mathematics Teaching. Willard Beatty, Commissioner of Indian Affairs, Department of Interior
3. Mathematics in Progressive Education. M. L. Hartung, Progressive Education Association
4. New Trends in the Teaching of Arithmetic. J. W. Studebaker, U. S. Commissioner of Education

SATURDAY, 7:30–8:15 A.M.

Group Breakfasts (Watch bulletin board for announcements)

SATURDAY, FEBRUARY 25, 8:30–10 A.M.
Coral Room

V. The Contribution of Progressive Education to Teaching Arithmetic

Presiding: John T. Johnson, Chicago Teachers College

1. Teaching Arithmetic in a Progressive Program. A. C. Rosander, U. S. Bureau of Labor Statistics
2. Discussion Panel. F. K. Knight, Purdue University, Lafayette, Indiana; H. O. Gillet, Principal, University of Chicago Elementary School.; F. S. Breed, University of Chicago

SATURDAY, FEBRUARY 25, 8:30–10 A.M.
Petit Cafe

VI. Secondary School Mathematics (Geometry and Logical Thinking)

Presiding: M. L. Hartung, University of Chicago

1. Panel Discussion. Alma Bowen, Shaker Heights, Ohio; Julius H. Hlavaty, High School of Science, New York City; H. Fawcett, University High School, Columbus, Ohio; Gilbert Ulmer, University of Kansas; Leroy Schnell, Indiana State Teachers College, Indiana, Pennsylvania

SATURDAY, FEBRUARY 25, 8:30–10 A.M.

VII. Radio, Movies, and Other New Features of Mathematics Teaching

Presiding: Hale Pickett, Ohio University, Athens, Ohio

1. Project Work in Geometry. Eloise Voorheis, Toledo, Ohio.

2. New Material and Equipment for Teaching Mathematics. Bjarne Ullsvik, University of Wisconsin High School, Madison, Wis.
3. Teaching Arithmetic by Radio—A Demonstration. H. E. Grime, Supervisor of Mathematics, Cleveland, Ohio

SATURDAY, FEBRUARY 25,
10:15–11:45 A.M.
Grand Ballroom

VIII. How to Make Junior High School Mathematics Function, Mathematics Club of Greater Cincinnati
Presiding: R. G. Wilbur, Cincinnati
1. Discussion Panel. Louis Strum, Glendale, Ohio; J. V. Naugle, Wyoming, Ohio; Marie Hall, Reading, Ohio; Rhesa Newlin, Withrow High School, Cincinnati, Ohio; Frieda E. Liebing, Columbian, Cincinnati, Ohio; Gladys Kloak, Hartwell High School, Cincinnati, Ohio.

SATURDAY, FEBRUARY 25,
10:15–11:45 A.M.
Directoire Room

IX. Junior and Senior High School
Presiding: H. E. Grime, Supervisor Mathematics, Cleveland
1. Social Mathematics in High School. J. M. Jacobs, Cleveland, Ohio
2. Social Mathematics in High School. W. O. Smith, Cleveland, Ohio
3. Mathematics Applied in the Study of Social Problems. Howard Whipple Green, Cleveland, Ohio
4. Discussion leader. C. L. Thiele, Detroit, Mich.

SATURDAY, FEBRUARY 25,
12:15–1:45 P.M.

X. The Place and Status of Mathematics
Luncheon: Delegates, Representatives and Directors
Reports on Status of Mathematics in various states.
Brief Addresses: William E. Betz, W. D. Reeve

SATURDAY, FEBRUARY 25, 2–4 P.M.
Petit Cafe

XI. Research Findings in Arithmetic, Joint Meeting with A.E.R.A.
Presiding: William A. Brownell, Duke University, Durham, North Carolina
1. Relative Difficulties of Types of Examples in Long Division with Two-Figure Divisors. Leo J. Brueckner, University of Minnesota
2. Consistency of Error in Division. Foster E. Grossnickle, State Teachers College, Jersey City, N. J.
3. A Comparison of Three Instructional Methods in Problem Solving. C. L. Thiele, Detroit Public Schools
4. Qualitative Analysis of Some Imponderables in Arithmetic Teaching. Harry G. Wheat, West Virginia University, Morgantown, W. Va.
5. The Incidence of Learning the Fundamentals of Arithmetic at Different Grade Levels. G. M. Wilson, Boston University

SATURDAY, FEBRUARY 25, 2–4 P.M.
Grand Ballroom

XII. New Developments in Secondary School Mathematics
Presiding: Raleigh Schorling, University of Michigan, Ann Arbor, Mich.
1. The Report of the Joint Commission, by K. P. Williams, Chairman of the Commission, University of Indiana, Bloomington
2. How the Report Can Be Improved. E. R. Breslich, University of Chicago; Martha Hildebrandt, Maywood, Illinois; L. C. Karpinski, University of Michigan; Vera Sanford, State Normal School, Oneonta, New York

SATURDAY, FEBRUARY 25, 2–4 P.M.

XIII. Visual Aids in Mathematics
Presiding: E. W. Schreiber, State Teachers College, Macomb, Illinois
1. Home Made Visual Aids. Edith Woolsey, Sanford Junior High School, Minneapolis, Minn.
2. Showing of Films Available.

SATURDAY, FEBRUARY 25, 4–5 P.M.
Coral Room

XIV. Business Meeting
Presiding: H. C. Christofferson. Open to all members of the Council

SATURDAY, FEBRUARY 25, 6:30–10 P.M.
Rainbow Room

XV.
Discussion Dinner (See Special Program)
6:00–8:00
Music by Cleveland Schools
8:00–8:15
Address: Mathematics in Civilization
8:15–9:00
F. R. Moulton, Secretary American Association for Advancement of Science.

Smithsonian Institute, Washington, D. C.

SPECIAL NOTE: Discussion from Floor is cordially invited. At every meeting, except the general meetings Friday and Saturday evening, opportunity will be given for questions and comments from the floor. Active participation by listeners during these periods for discussion is not only invited but urged so as to make the meetings more valuable to all.

Hosts and Topics for the Discussion Dinner
Saturday, February 25, 6:30–10 P.M.

Table Number	Host	Topic for discussion
1.	Miss Kate Bell	Motivating and Illustrating Mathematics Teaching
2.	E. R. Breslich	Mathematics for Everybody at the Ninth Grade Level
3.	W. A. Brownell	The Learning Process in Arithmetic
4.	John P. Everett	Topic To Be Announced
5.	Harold Fawcett	Mathematics in General Education
6.	Martha Hildebrandt	Discovery—Its Place and Importance in Teaching Math.
7.	M. L. Hartung	Testing Achievement of "Intangibles" in Mathematics
8.	Julius H. Hlavaty	The Place for Demonstrative Geometry in the Secondary School Curriculum
9.	John T. Johnson	Can Mathematics Be Taught Progressively in a "Subject-Matter" Curriculum
10.	Ruth Lane	Adjusting Ninth Grade Mathematics to Differences in Ability and Preparation
11.	Virgil S. Mallory	Some Questions in the Joint Commissions Report and Their Effect on High School Teaching
12.	Hale Pickett	Topic To Be Announced
13.	Vera Sanford	"Let's just Talk, No Assigned Topic"
14.	W. S. Schlauch	Sciences a Mathematics Teacher Should Know
15.	Raleigh Schorling	Mathematics for the Backward Pupil
16.	E. W. Schreiber	Topic To Be Announced
17.	L. H. Whitcraft	Junior High School Algebra for the Child not Going to College
18.	Edith Woolsey	Sources and Uses of Up-to-date Materials for Socialized Mathematics.

Reservations should be sent to Mr. Jesse J. Rush, 12962 Emerson Avenue, Lakewood, Ohio.

Tickets $2.00 may be obtained at any time before noon on Saturday.

EDITORIALS

On Postponing Algebra and Geometry

In an article by Wm. B. Brown, Director, Secondary Curriculum Section, of the Los Angeles City School District in the June 1938 issue of the *Sierra Educational News* on "Training for College" appears the following:

> Arithmetic is Improving
>
> Definite improvement in preparatory work in college mathematics is indicated. Algebra is quite generally being deferred to the senior high school, inasmuch as pupils tend to meet with greater success in the study of this subject at higher maturity levels. Furthermore, there is indication of a stronger foundational arithmetic program in the junior high school. Evidence points in the direction of three years of general mathematics in grades 7, 8, and 9, closely related to problems and commercial realities of everyday life.
>
> In the senior high school some experimenting is being planned with a view to determining more effective ways of teaching algebra and geometry. No definite outcomes have been indicated as yet but there are possibilities of humanizing these subjects and of correlating the work of algebra and geometry in ways which we have not anticipated in the past. As with foreign languages there is serious question as to whether algebra and geometry provide a preparation for college which cannot be obtained in other ways. It is our prediction that the day is not far distant when algebra and geometry will not be required for admission to liberal arts colleges.

It is clear that the idea held by Mr. Brown that algebra is too hard for junior high school pupils is shared by many other educators, particularly those who are not especially interested in the mathematics program. The notion that algebra and geometry should be "pushed up" if followed to any large degree, really means "pushing these subjects out" altogether.

It is not necessary here to argue the question as to whether algebra and geometry should be required of all applicants for admission to liberal arts colleges. It is important to point out that the solution to the problem of having pupils profit by a study of algebra is to have them begin the study of the subject earlier, *not later* than has been the custom. Of course, a great deal depends upon what is taught in algebra. Mr. Brown admits that general mathematics is probably due to occupy the time of the children in grades 7, 8, and 9. If then we can begin by a study of some of the simple formulas, equations, and graphs in the seventh grade and build on this basis a superstructure of substantially good algebra that the well educated citizen should know, we need not worry about the college entrance feature in most schools.

Similar remarks hold with reference to geometry. If an informal type of geometry can be started in the seventh grade and can be carried on through the eighth and ninth grades, a better basis for the demonstrative geometry in the tenth grade will be made. Here again the content of this informal geometry must be carefully chosen.

In this connection, it may be interesting to see what Mr. Merton E. Hill, the Director of Admissions at the University of California and past President of the Southern Section of the California Teachers Association thinks of the importance of mathematics. In an article in the April 1938 issue of the *Sierra Educational News* on "Secondary School Training," he said

> Some educators get their greatest satisfaction in an attack upon some procedure, or course, or subject, or objective. Some would rule out elementary mathematics, and language, and science, not realizing that they are making an attack on the scholarly training of students; not realizing that mathematics presents a way of thought and that it stands above everything else for exactitude of performance and quantitative thinking; they fail to realize that lan-

guage unlocks a vast storehouse of knowledge and that laboratory sciences present a way of doing that requires rigorous thinking.

Neither high school nor junior college will ever perform its mission if it permits requirements to creep into its curriculums that ignore qualitative and vigorous thinking so basic in the intensive struggle that every new worker will encounter. It is not avoidance of subjects that educators should advocate, it is better teaching, and this can be effected by constant application to life conditions.

These two opposing opinions coming from two educators in the same state reflect pretty well the difference in points of view as to the place of mathematics in secondary education in this country. It is the business of those interested in the future of mathematics education to see to it that the second view is more widely held.

W.D.R.

Gifted Pupils

ONE of the most important things for teachers to consider is the value of discovering the gifted pupils in their schools. In one of the New York City schools where attention is being given to individual differences in ability and particularly to the encouragement of pupils of scholarly minds, an incident occurred recently that is worthy of notice. One of the Assistant Superintendents, Dr. Greenberg, was making a visit to this school and noticed that a group of pupils who were studying art were discussing Archimedes. He thereupon inquired as to why an art class should be concerned with Archimedes. A boy (a gifted pupil with a very strong attachment for mathematics) who had spoken previously to the art class about Archimedes was not present at the time, but upon hearing of the question wrote Dr. Greenberg the following letter:

Several of my classmates have informed me that you questioned them as to the relationship of Archimedes to our unit on "Music, Art and Literature." We are not confining ourselves to "Music, Art and Literature," but are also studying the background that helped to produce this culture. We feel that the only way that we can acquire a full view of this is to study contemporary contributors to the advance of civilization. I being greatly interested in mathematics volunteered to deliver a report on Archimedes who was famous for his mathematical research.

But this is not the only way Archimedes is related to our unit on "Music, Art and Literature." In the act of writing any great piece of music a knowledge of mathematics is essential. Also in any good work of art it enters in the form of perspective without which a drawing is apt to be void and lifeless. Therefore, Archimedes has been included in our unit.

I hope you will soon visit our classroom again for all the children enjoy the talks you often give them.

This is certainly another interesting manifestation of how much we may expect from gifted pupils when they are treated as though we thought them able to do good work. This particular boy has the mental age of a junior in college, and yet he is only in the ninth grade in school.

W.D.R.

Annual Luncheon

THOSE who wish to attend the Luncheon Meeting for Delegates and Representatives on February 25, at noon, in the Hotel Carter, Cleveland, should send reservations to Mrs. Florence Brooks Miller, Chairman of State Representatives, 3295 Avalon Road, Shaker Heights, Ohio. This luncheon is part of the program of the Twentieth Annual Meeting of the National Council of Teachers of Mathematics, which will be held in Cleveland, Ohio, February 24 and 25.

IN OTHER PERIODICALS

By NATHAN LAZAR
Alexander Hamilton High School, Brooklyn, New York

1. Addelston, Lorraine W., "Shall We Teach to Test, or Test to Teach?" *High Points*, vol. 20, no. 9, pp. 22–26. September, 1938.

This is the first of a series of reports to be presented by the Test and Measurements Committee of the recently organized Association of Teachers of Mathematics of New York City.

The difficulties encountered by the conscientious teacher in making a test, in scoring it, and in interpreting it are interestingly presented. "A test is valuable and worth the time spent on it if, and only if, it is an instrument of teaching. . . . A score must be recorded quantitatively, but it must be analyzed qualitatively for remedial purposes. Unless the test shows wherein the teacher and the pupil have failed to attain their common goal, it has been no test, but, instead, a hurdle. Unless the teacher makes a conscious effort to point out the deficiencies and to remedy them, he is no teacher, but an arbitrary demi-god."

A promise is held out for a more intensive and extensive study of the problem.

2. Boyer, Lee Emerson, "Uses of Certain Topics in Algebra." *School Science and Mathematics*, 38: 921–923. November, 1938.

In this article the writer satisfies a long felt need for a list giving the specific uses for certain algebraic concepts and techniques.

The following is a list of seven topics and the number of applications cited:

a. Imaginary Numbers—6.
b. Systems of Linear Equations—4.
c. Systems of Quadratic Equations—3.
d. Graphs of Hyperbolas—5.
e. Significance of Determinants—3.
f. Roots of Cubic and Quartic Equations —5.
g. Inequalities—5.

After enumerating the applications of the above topics, the writer makes the following significant remark: "This use for algebra is perhaps not so obvious to the uninitiated in algebra, but in the minds of forward looking educators it becomes a significant objective for the well educated citizen of tomorrow."

3. Gorski, T. M. and Powers, William H., "The First Course in Algebra." *National Mathematics Magazine*, 13: 34–36, October, 1938.

The authors believe that the first college course in algebra needs revision downward in the number of topics taught and upward in the time spent on enrichment material. They, therefore, sent a form containing all the topics listed in the nine texts on *First College Algebra* to 56 institutions, asking the departments of mathematics to classify the items listed as essential, desirable, and undesirable, for the course.

As a result of the replies received they make the following conclusions:

(1) "The first course in college algebra may be revised to include 85 subtopics without sacrificing material deemed essential by 63 per cent of representative reporting institutions.

(2) "Revision will permit time for additional topics at the discretion of the instructor and will permit the introduction of more original problems, historical material, and some elementary treatment of the philosophy of numbers."

The essential and non-essential topics are indicated by means of a table and a graph.

4. Löf, John L. C., "The Conic Compass." *School Science and Mathematics*, 38: 842–846, November, 1938.

The author describes the theory, the mechanical construction, and the operation of an instrument that "can draw any conic section with any eccentricity, and in particular any specific curve desired. However, its greatest opportunity lies in the demonstration of the properties of the conics and illustration of how the curves are obtained from the cone."

Two diagrams are included.

5. Mitchel, U. G., "The Study and Teaching of the History of Mathematics." *National Mathematical Magazine*, 13: 22–29, October, 1938.

A discussion of the importance of the history of mathematics for the teacher of mathematics.

6. Mueller, Felicite M., "Pythagoras." *School Science and Mathematics*, 38: 846, November, 1938.

A sonnet by a high school student from She-

boygan, Wisconsin. Very cleverly it incorporates within the traditional compass of fourteen lines the birthplace, the wanderings, and the accomplishments of the ancient mathematician.

This sonnet appeared also in *The Mathematics Teacher*, 31:342, November, 1938.

7. Porges, Arthur, "An Interesting Application of the Algebraic Method of Balancing Chemical Equations." *School Science and Mathematics*, 38: 853–854, November, 1938.

The author first describes the algebraic method of balancing chemical equations, which is better known among teachers of chemistry than among teachers of algebra.

He then calls attention to an interesting application of this method which, he believes, has nowhere been mentioned.

"If a chemical equation be mistakenly written in such a form that two compounds on opposite sides of the equality sign are interchanged, the error, which may not be obvious from a chemical viewpoint, is easily detected and corrected when algebra has been applied."

8. Powers, F. R. and Engle, A. W., "Must *All* Mathematics Be Forgot?" *School Science and Mathematics*, 38: 871–888, November, 1938.

A brief review of some fundamental mathematical processes that students in the upper reaches of the high school find difficult.

The authors conclude that "the bases of recall are (1) a thorough understanding of processes—not just a knowledge of their routine, (2) similarities standing out more than differences, (3) new processes learned in terms of old familiar ones where possible. The whole history of mathematics is one of finding easy, accurate, and reasonable ways of doing things. We believe that if we are able to teach our students to understand a few basic processes rather than a jumble of formulas, we are teaching them to think in mathematics. Then recall becomes the least of our worries."

9. Sosnow, George, "About Failures in Mathematics." *High Points*, vol. 20, no. 5, pp. 77–78, May, 1938.

"Occasionally statistics appear which show, in a very general way, that the number of failures in mathematics is somewhat larger than in most other subjects. It is not ordinarily mentioned, however, that this lack of success in mathematics prevails mostly in elementary algebra. In the later terms, and especially in the more advanced grades, the number of failures in mathematics is quite small."

The author ascribes this excessive failure to the absence of any pre-requisite for elementary algebra, and concludes that "it is really an injustice to the pupil to compel or to advise him, directly or indirectly, to include elementary algebra in his program after he has furnished reasonable evidence of unsuitability for this work."

10. Tannenbaum, Adolph, "Whither Geometry?" *High Points*, vol. 20, no. 9, pp. 43–44, September, 1938.

The author gives an amusing but sympathetic account of the mental tribulations that an average student undergoes while trying to understand and to reproduce an indirect proof. He believes that "there are large numbers of boys who in mathematics are slow, even if not sure. Extending the present course over three terms so that one full term could be devoted to Book I might meet their needs. A second alternative is to make the present course more digestible by shifting the emphasis away from formal demonstration. . . . Isn't it good mathematics to substitute less rigor and more understanding for more rigor and less understanding?"

11. "What Value the Teacher-in-Training?" (Anonymous.) *High Points*, vol. 20, no. 9, pp. 49–52, September, 1938.

It has been the custom of the chairman of one of the mathematics departments of the New York City high schools to ask the teacher-in-training, when his service was about to expire, to give him a statement as to what he thought he got out of his experience in that position and what suggestions he had to offer that might make the future training work of the supervisor more efficient. The statement made by a teacher-in-training to an anonymous chairman is reproduced in this article and should make interesting reading to the mathematics supervisor as well as to the experienced and inexperienced teacher.

A Good Story

A recent Michigan story tells of an interesting and valuable mathematics course that had been worked out for a group of children classified by the school as distinctly less able than the ordinary. It was such a good course that "the brightest boy in school" became interested and wished to take it. The school authorities hesitated to say in so many words that the course was intended for those who ranked low in mental ability, but they did manage to make clear that the course was not intended for him. "Say," said the boy, "how dumb does a fellow have to be to take this course?"*

* Ryan, W. Carson: *Mental Health Through Education*. New York, The Commonwealth Fund, 1938. P. 134.

NEWS NOTES

Boys with special talents for work in physics and biology have been selected for enrollment in New York City's Bronx High School of Science. An attempt is being made here to develop "the scientific way of thinking" and to rear the school's three hundred boys in the footsteps of Pasteur, Faraday, Einstein, and Gibbs.

Forty per cent of the students' time is spent on scientific studies, the remaining time on cultural subjects. Even in the latter scientific ideas will be met in the writing of English and applications in the social studies. The school has the best of modern laboratories, its own power plant and shops.

Three hundred boys were admitted this fall, according to Principal Morris Meister. Six hundred more will be admitted next year. Eventually the school will have about twenty-five hundred students. Candidates for admission go through a strenuous program of testing and evaluating; and it is supposed that the student body includes a number of potential geniuses.

Qualifications for the faculty are also very high. The present staff of twenty-five will be increased to one hundred within the next few years. Only experts and specialists in certain fields are to be chosen.

The Association of Teachers of Mathematics of New York City met December 10 at 10:30 A.M. in the Conference Room of the New York Times Building. The agenda included an address by Joseph Jablanower, of New York City's Board of Education Board of Examiners, on "Essentialism and Progressivism in the Teaching of Mathematics." Barnett Rich, of the New York High School of Music and Art, gave a demonstration and talk on "Certain Logical Aspects of Geometry."

The Association of Teachers of Mathematics of the Middle States met Saturday morning, November 26 as part of its special activities in the 52nd Annual Convention of The Middle States Association of Colleges and Secondary Schools and Affiliated Associations in Atlantic City, November 25 and 26.

The following addresses were given: "Alpha, Beta, and Gamma Examinations—Expectations and Results," J. R. Kline, Professor of Mathematics, University of Pennsylvania, and Reader for the College Entrance Examination Board; "Broadening Mathematics for the Bright Pupils," J. Whitney Colliton, Central High School, Trenton, N. J.

President of the Association J. T. Rorer, William Penn High School, Philadelphia, and Secretary Ruth Wyatt, Wilson Junior High School, Philadelphia, conducted the business meeting.

The 1938 annual meeting of the Department of Mathematics of the Missouri State Teachers Association began with an informal luncheon in the Pickwick Hotel, Kansas City, Friday, November 18.

After an interlude of instrumental music by a brass ensemble from the Kansas City Southeast High School Band, Chairman Robert E. White introduced Professor U. G. Mitchell, University of Kansas, who supplemented his "Stereopticon—Glimpses into the History of Mathematics" by interesting comments, and Professor George H. Jamison, Northeast Missouri State Teachers College, who expressed convictions in "Neglected Mathematics in the Teaching of Arithmetic."

Vice-Chairman, Gaylord C. Montgomery, explained and urged every mathematics teacher to make an affiliation with the National Council of Teachers of Mathematics, citing articles in THE MATHEMATICS TEACHER of condensed research-interest to every busy teacher.

In the business meeting the following officers were elected by acclamation for 1938–39: Chairman, Gaylord C. Montgomery, John Burroughs High School, Clayton; Vice-Chairman, Nelle Kitchens, Hickman High School, Columbia; Secretary-Treasurer, Paul A. De Vore, Central Missouri State Teachers College.

At the meeting of the Mathematics Section of the Arkansas Education in the Little Rock, Ark., Senior High School, the following program was presented:

"The Use of Projects in the Teaching of Mathematics in High Schools," Mrs. Elsie Cummings, Hot Springs. She described in detail some of the projects she has used, including the student publication "The Third 'R'."

"The Value of Teaching Mathematics in High School," Mrs. L. M. Caraher, Arkansas State Teachers College. Mrs. Caraher brought out interesting methods of teaching certain parts of the high school courses.

New officers elected were: President, Miss Kathrine Mires, Augusta, and Secretary, Miss Mildred Horton, Forrest City. The retiring president was Paul Brown, Lonoke, and the retiring secretary Miss Mildred Lord, Little Rock.

The Central Association of Science and Mathematics Teachers held its thirty-eighth annual convention at the La Salle Hotel in Chicago, November 25 and 26.

"The Program in Science for 1950" was the topic upon which Dr. Morris Meister, of the Bronx High School of Science, New York City, addressed the Friday morning meeting. He was followed by Dr. Francis O. Schmidt, Professor of Zoology, Washington University, St. Louis, with "Recent Applications of Physics and Chemistry in Biology." "Josiah Willard Gibbs, an American Scientist and Mathematician," by Professor Rudolph E. Langer, University of Wisconsin, closed the morning session.

Dr. J. O. Perrine, Associate Editor of the *Bell System Technical Journal*, New York City, gave a demonstration lecture on "Words, Wires, Waves" at the reception and annual dinner.

After the business meeting Saturday morning, another demonstration lecture was given on "How to Improve Science and Mathematics Teaching Through Better Reading" by E. A. Taylor of the American Optical Company, Southbridge, Mass., and Dr. Louise Farwell-Davis of the National College of Education, Evanston, Ill. Group meetings followed.

Elementary School Group, D. W. Russell, presiding.

"Demonstration of a Radio Lesson in Elementary Science," Mary Melrose, Supervisor of Elementary Science, Cleveland, Ohio, assisted by Veva McAtee, Supervisor of Science in Hammond, Indiana, and a class of pupils from the Hammond Schools.

10:45—Panel Discussion—"Controversial Issues Related to the Teaching of Science in the Elementary Grades," Paul G. Edwards, Chairman, Chicago Public Schools, Chicago, Ill.; Dr. Walter A. Anderson, Northwestern University, Evanston, Ill.; Clara Belle Baker, Director of the Children's School, National College of Education, Evanston, Ill.; Dr. W. C. Croxton, State Teachers College, St. Cloud, Minn.; Mildred Fahy, Principal of the Peirce School, Chicago, Ill.; Dr. Louis M. Heil, University of Chicago, Ill.; Lillian Hethershaw, Drake University, Des Moines, Iowa; Ellis C. Persing, Western Reserve University, Cleveland, Ohio; Dr. E. T. McSwain, Northwestern University.

Junior High School Group, T. J. Kuemmerlein, presiding.

"Science Concepts To Be Taught in the Junior High School," Dr. Morris Meister, Bronx High School of Science, New York City; "A Program for Conservation Education in the Junior High School," B. J. Rohan, Superintendent of Schools, Appleton, Wis., and Guy Barlow, Principal of Wilson Junior High School, Appleton; "Mathematical Concepts To Be Taught in the Junior High School," Dr. J. S. Georges, Wright Junior College, Chicago.

Senior High School Group, Lillian Bondurant, presiding.

"Some Mathematical Concepts To Be Taught in the Tenth Grade," H. C. Christofferson, Professor of Mathematics, Miami University, and President of the National Council of Teachers of Mathematics; "Popular Beliefs That Are Not So," W. W. Bauer, Director of the Bureau of Health and Instruction, American Medical Association; "A Program in Science for All of the Pupils of the Senior High School," Sherman R. Wilson, Head of the Science Department, Northwestern High School, Detroit, Michigan.

Junior College Group, J. M. Kinney, presiding.

"The Desirability of the Understanding of Mathematical Concepts and Principles as a Preliminary Step to the Mastery of Manipulation and Technique," O. M. Miller, Woodrow Wilson Junior College, Chicago, Illinois; "Some Aspects of Laboratory Work in General Chemistry," Dr. H. I. Schlesinger, Professor of Chemistry, University of Chicago; "Attitudes Related to the Study of College Science," P. P. Dewitt, Hanly Junior High School, University City, Missouri; "The Function of Biological Science in General Education," R. Clark Gilmore, Wright Junior College, Chicago, Illinois.

The Men's Mathematics Club of Chicago and the Metropolitan Area climaxed an active half year with their December dinner meeting on the sixteenth. The mathematics faculty of the Lane High School contributed the program which included the following topics:

"Mathematics Assistance Room," "Probable Errors," Vocational Mathematics," "Arithmetic of Non-Decimal System," "Handling Disciplinary Cases in Co-operation with Counseling Department," and "How Shall We Revise the Mathematics Courses."

The Oak Park School faculty gave the program for the November 18 meeting. The subjects discussed were "Motivating High School Mathematics," "Checks on Classification," "Facts about My Algebra Special Class," "How I Would Like to Teach Geometry," "Our Duty as Teachers of English," "On Your Marks," "Selecting New Problem Material in Algebra," and "Mooseheart Experiences."

At the October Meeting, W. H. Clark spoke on the subject "A Brief Survey of the Amount of Kind of Mathematics Offered Commerce and Administration Students in the Junior Colleges

of the United States." Mr. Clark wrote the greater part of his paper while in California on a sabbatical leave of absence from the University of Southern California.

John Schommer, Athletic Director, Professor of Industrial Chemistry, Placement Director, and Trustee of Armour Institute of Technology, entertained the group with an address on "Football."

J. A. Nyberg distributed the second part of the Preliminary Report of the Committee on the Place of Mathematics in the Secondary School.

STATE REPRESENTATIVES OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

1. Alabama: J. Eli Allen, Phillips High School, Birmingham
2. Arizona: Miss Myra R. Downs, 93 W. Culver St., Phoenix
3. Arkansas: Dr. Davis P. Richardson, Univ. of Arkansas, Fayetteville
4. California: Miss Emma Hesse, Univ. H. S., Oakland, Cal.
5. Colorado: H. W. Charlesworth, East H. S., Denver
6. Connecticut: Miss Dorothy S. Wheeler, Bulkeley H. S., Hartford
7. Delaware: Edwin H. Downs, Dover, Delaware
8. Dist. of Col.: Mrs. Ethel Harris Grubbs, 751 Fairmont St., Washington, D.C., and Miss Veryl Schult, Wardman Park Hotel, Washington, D.C.
9. Florida: Dr. F. W. Kokomoor, Univ. of Florida, Gainesville
10. Georgia: Dr. Eucebia Schuler, 312 W. Church St., Americus, Ga.
11. Idaho: H. Main Shoun, Supt. of Schools, Jerome, Idaho
12. Illinois: C. M. Austin, Oak Park, Ill.
13. Indiana: Dr. L. H. Whitcraft, Ball State Teachers College, Muncie
14. Iowa: Miss Dora Kearney, Iowa State Teachers College, Cedar Falls, Iowa
15. Kansas: Dr. U. G. Mitchell, Univ. of Kansas, Lawrence, Kansas
16. Kentucky: Miss Dawn Gilbert, 1331 Clay St., Bowling Green, Ky.
17. Louisiana: Mrs. H. L. Garrett, Box 327, Univ. Station, Baton Rouge, La.
18. Maine: Miss Pauline Herring, 360 Spring St., Portland, Maine
19. Maryland: Miss Agnes Herbert, Clifton Park Jr. H. S., Baltimore, Md.
20. Massachusetts: Harold B. Garland, 129 Houston Ave., Milton, Mass.
21. Michigan: Duncan A. S. Pirie, 950 Selden Ave., Detroit, Mich.
22. Minnesota: H. G. Tiedeman, Mountain Iron, Minn.
23. Mississippi: Dewey S. Dearman, State Teachers College, Hattiesburg, Miss.
24. Missouri: G. H. Jamison, State Teachers College, Kirksville, Mo.
25. Montana: Miss Gertrude Clark, 403 Eddy Ave., Missoula, Mont.
26. Nebraska: Dr. A. R. Congdon, Univ. of Nebraska, Lincoln, Nebr.
27. Nevada: Miss Bertha Knemeyer, (Located at Chazy Rural School, Chazy, in New York State now.)
28. New Hampshire: H. Gray Funkhouser, Phillips Exeter Academy, Exeter, N.H.
29. New Jersey: Charles D. Wildrick, Englewood, N.J.
30. New Mexico: Miss Olive Whitehill, 402 West 8th St., Deming, N. M.
31. New York: H. C. Taylor, Benj. Franklin H. S., Rochester, N.Y.
32. North Carolina: Prof. H. F. Munch, Chapel Hill, N.C.
33. North Dakota: Miss Henrietta L. Brudos, State Teachers College, Valley City
34. Ohio: Dr. Hale Pickett, Athens, Ohio
35. Oklahoma: Miss Eunice Lewis, 208 S. Birch, Sapulpa, Okla.
36. Oregon: Edgar E. DeCon, Univ. of Oregon, Eugene, Ore.
37. Pennsylvania: Dr. C. N. Stokes, Temple Univ., Philadelphia, Pa.
38. Rhode Island: W. Wilson Talley, Mary C. Wheeler School, Providence, R. I.
39. South Carolina: Miss Hortense Rogers, Winthrop College, Rock Hill, S.C.
40. South Dakota: Miss Josephine Wagner, Sioux Falls Schools, Sioux Falls
41. Tennessee: Dr. F. L. Wren, Peabody College, Nashville, Tenn.
42. Texas: Miss Elizabeth Dice, North Dallas H. S., Dallas, Texas
43. Utah: Edwin M. Bronson, West High School, Salt Lake City, Utah
44. Vermont: C. H. Nicholson, 328 North Ave., Burlington, Vt.
45. Virginia: Dr. L. G. Lankford Jr., Univ. of Virginia, University, Va.
46. Washington: Miss Kate Bell, The Lewis and Clark H. S., Spokane, Wash.
47. West Virginia: Miss May L. Wilt, 107 High St., Morgantown, W.Va.
48. Wisconsin: Elmer Schuld, Wisconsin High School, Madison, Wis.
49. Wyoming: Dr. O. H. Rechard, Univ. of Wyoming, Laramie, Wyo.

NEW BOOKS

Excursion in Mathematics. By Ernest R. Breslich. The Orthovis Company, 1938. 47 pp. Price, 36¢.

This interesting new monograph treats the basic facts of geometry in such a way as to aid the teacher in giving the pupil a knowledge of these facts so as to equip him better to meet the needs of his daily life. One of the difficulties so often encountered in the teaching of geometry is that we live in a world of three dimensions and teach only the geometry of Flatland. Professor Breslich has sought to remedy that situation in his book by showing geometric figures, both as they appear on the flat page of the textbook and as they are actually observed "in familiar settings, and practical situations, in art and nature, in designs and landscapes, and in construction work." In this way the pupil is helped to visualize better the important spatial relationships that are involved. To facilitate the understanding of these relationships an instrument called the "arthoscope" is furnished with the book. Teachers of mathematics should see this new publication.

W.D.R.

Remedial Arithmetic. (For High School Pupils.) By Allan R. Congdon and Ronald B. Thompson. The University of Nebraska, 1938, vii+212 pp. Price, including a manual for teachers and a set of tests, $1.30. A 20% discount is allowed if 5 or more sets are ordered. Replacement tests can be secured for 40¢ each or 32¢ when ordered in quantities of 5 or more.

The purpose of this course for high-school pupils is two-fold. It is planned to discover the weaknesses of the pupils and correct them and to prepare the pupil generally to meet his arithmetical needs adequately in daily life problems.

It is obvious that pupils leaving high school today are inadequately equipped in arithmetic, and some kind of training needs to be devised for such pupils. It will be of interest to see how this material can help to furnish the necessary remedial instruction.

The books are carefully and neatly prepared and are not expensive as to be prohibitive in many schools.

W.D.R.

Theory of Equations. By Joseph Miller Thomas. McGraw-Hill Book Company, 1938, x+211 pp. Price, $2.00.

This new book is designed as a textbook for a one-term course in the theory of algebraic equations for both undergraduate and graduate students. It is the author's intention to treat the topics in a manner which, although elementary, forms a natural approach to the Galois theory and other phases of algebra to be studied later by the specialists.

The book begins with a treatment of the elementary properties of numbers, permutations and determinants, followed by chapters on systems of linear equations, polynomials in a single indeterminate, graphical methods, roots of unity, single equation in a single unknown, symmetric functions, constructibility, resultants and discriminants, and simultaneous systems.

Certain special features of this book will be of interest to prospective users of this text. In view of the fact that few texts of this kind are available particularly for undergraduates, this book should have a good trial.

W.D.R.

A New Geometry. By A. W. Siddons and K. S. Snell. Cambridge: at the University Press; New York: The Macmillan Company; 1938, xvi+302 pp. Price, $1.32.

For several years now the teachers of geometry in the Mathematical Association of England have been trying to improve the teaching of Euclid. This new book by two prominent teachers in England will be of interest to the teachers in this country who have been watching the movement.

With solid geometry passing out of the schools as a separate subject, it will be of interest to teachers first to see how plane and solid geometry are presented in the same one-year course. They will be interested also to see how Euclid has been revised.

While the systematizing motive may not be as evident in the earlier parts of the book, the authors have arranged a section at the end of the book where the theorems are arranged in a systematic course, the proofs being written out in the form in which the pupil is expected to reproduce them.

W.D.R.

An Orientation in Science. By Eleven Members of the University of Rochester Faculty. Edited by C. W. Watkeys, McGraw-Hill Book Company, 1938, x+560 pp. Price, $3.50.

This is a new book that will be of interest and value particularly to those who are looking for suitable materials for general education such as is needed in some of our orientation courses in the colleges. Besides the introduction there are chapters on astronomy, geology, chemistry, physics, biology, paleontology, physiology, bacteriology, psychology, mathematics, and the scientific method.

The book is neatly and carefully prepared, is well illustrated and ought to be helpful in giving to the non-specialist and interested student the kind of general background in science which he needs. The book should be of interest to scientifically minded laymen as well as college students.

W.D.R.

American Wings. By Burr Layson. E. P. Dutton and Company, 1938, 214 pp. Price, $2.00.

This is really a book on modern aviation for the general reader. It has a foreword by Captain Eddie Rickenbacker in which he recommends the book. It is very generously illustrated and doubtless will be read by large numbers of people interested in modern aviation.

It is an informal story of flying in this country, including an account of the history, experiences, and achievements of aviation.

W.D.R.

Electrical Occupations for Boys. By Lee M. Klinefelter. E. P. Dutton and Company, 1937, 226 pp.

This book is intended to cover the entire field of electrical occupations available to boys when they grow up. It has a narrative thread running all the way through the book which makes it more interesting to the young boy reader and provides a setting for the information imparted that gives it added color.

The book should appeal particularly to those boys who have not had the opportunity in school to learn about things electrical. It may also conceivably be of help to some readers in choosing the type of occupation they wish to follow.

W.D.R.

The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry. By Nathan Lazar. The Mathematics Teacher, 525 West 120th St., New York City, 1938. 66 pages. Price, $1 postpaid.

The concepts to which Dr. Lazar refers in the title of his dissertation are the Converse, Inverse, Contrapositive and its extension and the Law of Converses. Each of these concepts is examined in the light of the history of its occurrence in elementary geometry, of the inadequacy of its previous definition and of its present and potential uses in a school course in geometry—the latter being well illustrated by numerous examples from the propositions of elementary geometry.

For example, the converse of a theorem is found to have been treated inadequately in text books and the writer gives prominence to his re-definition of the concept in terms of what he calls the multi-converse definition. Following the definition that to obtain converses of theorems we may interchange any number of conclusions with an equal number of hypotheses the author both illustrates the increase in the number of converses which a theorem may now possess, and shows the use to which their discovery by logical means may be put in exercising the mathematical imagination of pupils. A section on disguised converses shows the logical connection between the usual three congruence theorems—a result which is certain to be of interest to pupils who are familiar with them.

The same necessity for careful re-definition applies also to the inverse of a theorem, and this re-definition is given and is well illustrated by examples in the second chapter. The author, however, reserves for chapters III and IV, on the Contrapositive, his most interesting contribution. While not actually discovering the law of the contrapositive he sheds interesting light on its previous use in geometry by means of an historical section, and then proceeds to redefine it and again to illustrate the definition copiously. The definition as given is that "the Contrapositive of a theorem is another theorem obtained from it by contradicting both the hypothesis and the conclusion and by interchanging their respective places" (p. 37). An extended form of the theorem (p. 42) relating to theorems with one conclusion and a number of hypotheses is given and geometrical examples are added to show how many true propositions may be stated starting from a single proposition, if we accept the validity of the law of Contraposition. The author then makes the suggestion that this law should be introduced as an axiom of logic, and that pupils should be practised in its use, again to exercise mathematical imagination—to do in fact, and more logically, what the investigative exercise in geometry was introduced to do.

In like manner the Law of Converses is extended, and applied to situations in elementary geometry.

A notable feature of Dr. Lazar's study is the

striking manner in which he combines a scholarly treatment of the logical concepts he seeks to re-define with a judicious and apt selection of illustrations from the field of elementary geometry and, in later chapters, also from more geometrical material. By this apt use of illustration Dr. Lazar has chosen a most convincing way to enlist the interest of teachers of geometry in the new methods. Once enlisted, their interest is certain to carry them to a careful study of the local analysis itself.

I. S. TURNER

Proofs and Solutions of Exercises Used in Plane Geometry Tests. By H. Pickett, Bureau of Publications, Teachers' College, Columbia University Contributions to Education, No. 747. Price, $1.60

In this study Dr. Pickett sets out to determine a revised list of theorems in Plane Geometry. To this end he examines all examination papers in geometry for the years 1923–35 set by various public and state examining bodies in U.S.A. In all, 3002 questions from these papers are solved and then solutions are analysed to discover (a) the basic theorems, constructions, definitions, postulates and axioms, (b) the algebraic techniques and (c) the methods of proof used in their solution. The theorems, definitions etc. which are regarded by the author as basic are first tabulated, there being 125 theorems (based largely on those set out by the National Committee in the 1923 report), 19 postulates, 13 axioms, 18 constructions and 145 definitions in all.

The frequency of occurrence of these theorems, etc., in the solution of the 3002 examples is then determined and from this a percentile rating of theorems is given (Tables: pp. 14–25). The finding is that for solving the exercises set in these examination papers of the various examining boards, 71 theorems out of 125 fall in the 0–10 percentiles; 86 theorems fall in the 0–20 percentiles, and that only 4 theorems fall in the 80–100 percentiles. Regarding their use in solving this set of exercises as a criterion of reliability the author then asserts that "theorems known to have a greater utility should receive greater emphasis." The reader is tempted to investigate the 71 theorems in the 0–10 percentiles, and to compare them with their more fortunate brothers in the higher percentiles. The investigation is as illuminating as it is interesting. While admitting the utility of the "angle sum" and Pythagorean theorems (90 percentiles), one is concerned to find the theorem "angles in the same segment are equal" (0th percentiles) relegated to a position of minor importance compared with the theorem "In terms of a side s the area A of an equilateral triangle is $A = s^2/4\sqrt{3}$" (40th percentile).

The author gives further consideration to this list of theorems later, and in the meantime investigates to what extent the examining bodies (Regents, College Entrance Examination Board) set exercises which require the use of the theorems in their respective syllabi. He also examines the algebraic techniques used in solving these exercises, and the frequency of occurrence of the analytic, the indirect method and the method of superposition as the methods of proof necessary to solve these exercises. The answer is of course wholly in favour of the analytic method, with sufficient superposition present to show that its used as a method of proof dies hard, at least amongst examiners.

The author then returns to his original task of preparing a revised list of theorems. In addition to the already established criterion of utility, he gives three other criteria (use in a logical sequence, relative importance as indicated by lists of theorems in syllabi, persistence in representative texts) for establishing a revised list of theorems. The final revised list consists of 58 theorems which have remained after the application of the criteria mentioned. The list contains some surprises both of omission and of inclusion. The omissions are explained by saying that some of the theorems omitted (e.g. altitudes of a triangle meet in a point) may be introduced intuitively as postulates, others again as definitions, and still others as originals. This revised list of 58 theorems, together with the 19 postulates, 13 axioms, 18 constructions and 145 definitions selected for this study are regarded as basic to a study of demonstrative geometry in the senior high school.

I. S. TURNER

Important Reprint!

THERE is an ample supply of the reprint "The National Council Committee on Arithmetic," by R. L. Morton. This report, published in the October issue of THE MATHEMATICS TEACHER, was reprinted in answer to a demand by arithmetic teachers everywhere. They may be had for 10¢ each, or 5¢ each for quantities of a dozen or more.

D-26

Mathematics

FEB 9 1939

THE MATHEMATICS TEACHER

Volume XXXII FEBRUARY · 1939 Number 2

CONTENTS

		PAGE
Ancient Mathematics and the Development of Primitive Culture	William A. Cordrey	51
Mathematics in Herbart's Philosophy of Education	J. A. Lynch	61
The Effect of Remedial Work in Reading Comprehension upon Algebraic Achievement	Mildred I. Clark	65
Enrichment Materials for First-Year Algebra	David W. Starr	68
Criteria for the Evaluation of a Course of Study in High School Mathematics	Shirley E. Boselly, Jr., and Harriet Holbrook	78
Air Navigation and Secondary School Mathematics	John Kinsella and A. Day Bradley	82
Mathematics in Mechanical Engineering	George B. Karelitz	87
The Art of Teaching		
A Device for Teaching the Concept of the Trigonometric Functions	Laura Blank	88
In Other Periodicals		90
News Notes		93

PUBLISHED BY THE

NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

MENASHA, WISCONSIN : NEW YORK, N.Y.

Entered as second-class matter at the post office at Menasha, Wisconsin. Acceptance for mailing at special rate of postage provided for in the Act of February 28, 1925, embodied in paragraph 4, section 412 P. L. & R., authorized March 1, 1930.

THE MATHEMATICS TEACHER

Official Journal of the National Council of Teachers of Mathematics

Devoted to the interests of mathematics in Elementary and Secondary Schools
Editor-in-Chief—WILLIAM DAVID REEVE, Teachers College, Columbia University.
Associate Editors—VERA SANFORD, State Normal School, Oneonta, N.Y.
W. S. SCHLAUCH, School of Commerce, New York University.

OFFICERS OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS

President—H. C. CHRISTOFFERSON, Miami University, Oxford, Ohio.
First Vice-President—J. T. JOHNSON, Chicago Normal College, Chicago, Ill.
Second Vice-President—RUTH LANE, University High School, Iowa City, Iowa.
Secretary-Treasurer—EDWIN W. SCHREIBER, Western Illinois State Teachers College, Macomb, Ill.
Chairman of State Representatives—FLORENCE BROOKS MILLER, 3299 Avalon Road, Shaker Heights, Ohio.

ADDITIONAL MEMBERS ON THE BOARD OF DIRECTORS

One Year	R. R. BARBLICH, Chicago, Ill.	1939
	VIRGIL MALLORY, Montclair, N.J.	1939
	LEONARD D. HASSLER, Clayton, Mo.	1939
Two Years	WM. BETZ, Rochester, N.Y.	1940
	MARTHA HILDEBRANDT, Maywood, Ill.	1940
	EDITH WOOLSEY, Minneapolis, Minn.	1940
Three Years	KATE BULL, Spokane, Wash.	1941
	M. L. HARTUNG, Columbus, Ohio	1941
	D. R. SMITH, Springfield, Mass.	1941

The National Council has for its object the advancement of mathematics teaching in elementary and secondary schools. All persons interested in mathematics and mathematics teaching are eligible to membership. The membership fee is $2 per year and entitles each member to receive the official journal of the National Council—THE MATHEMATICS TEACHER—which appears monthly except June, July, August and September.

Correspondence relating to editorial matters, subscriptions, advertisements, and other business matters should be addressed to the office of the

THE MATHEMATICS TEACHER
525 West 120th St., New York City (Editorial Office)

Subscription to THE MATHEMATICS TEACHER automatically makes a subscriber a member of the National Council.

SUBSCRIPTION PRICE $2.00 PER YEAR (*eight numbers*)

Foreign postage, 50 cents per year; Canadian postage, 25 cents per year. Single copies 25 cents. Remittances should be made by Post Office Money Order, Express Order, Bank Draft, or personal check and made payable to THE MATHEMATICS TEACHER.

PRICE LIST OF REPRINTS

	4pp. 1 to 4	8pp. 5 to 8	12pp. 9 to 12	16pp. 13 to 16	20pp. 17 to 20	24pp. 21 to 24	28pp. 25 to 28	32pp. 29 to 32
100 Copies	$3.20	$4.65	$6.25	$6.60	$ 8.35	$ 9.80	$11.40	$12.15
100 Covers	3.20	4.80	7.40	8.15	10.85	12.20	14.25	15.30
Additional per C.	1.05	1.60	2.65	3.15	3.60	4.70	5.75	6.30

For 500 copies deduct 5%; for 1,000 copies or more deduct 10%.
Covers: For 100-50 covers, $3.00; additional 2½c each.
Plates printed on one side. For first 50 plates, $2.00; additional 1½c each. Printed on both sides, for first 50 plates, $3.00; additional 2½c each.
For pages above 32 pages add cost per schedule to make total. Example: for 44 pages add cost for 32 pages and 12 pages.
Covers: For covers requiring additional composition or changes in text or cover, an extra charge will be made.

Please mention the MATHEMATICS TEACHER when answering advertisements